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Multinational production and trade in an endogenous growth model with heterogeneous firms*

Hibret B. Maemir[†]and Thomas Ziesemer[‡]

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Abstract

This paper offers a unified framework to explore both the static and dynamic welfare effects of trade and multinational production (MP) in the presence of firm-specific productivity heterogeneity. The model captures the dynamic effects by allowing for R&D spillovers between firms in a framework of Helpman et al. (2004) that generates endogenous growth without scale effects. We show that multinational presence improves average productivity by strengthening the selection process among heterogeneous firms, but leads to a lower growth rate of intermediate varieties along the transition path toward the new steady state. Thus the presence of multinationals has an ambiguous effect on overall welfare. We also compare the welfare implications of a change in trade cost in our model and in trade models without multinationals. We find that the gains from trade can be higher or lower than the gains obtained in the trade-only models, depending on the degree of firm heterogeneity, the size of trade and FDI costs, and the magnitude of technology spillover parameters. We further show that firm heterogeneity always magnifies average productivity, international spillovers and fixed costs of developing a new variety, which leads to ambiguous effects on overall welfare. Calibrating the model to the US economy suggests that aggregate welfare improves in response to a reduction in trade and FDI costs for empirically plausible parameter values.

Keywords: firm heterogeneity, endogenous growth, trade, multinational production, technology spillovers.

JEL classification: F12, F23, F43, O31, O41

1 Introduction

Since the seminal work of Melitz (2003), much effort has been devoted to understanding the properties of heterogeneous firm models. A vast theoretical and empirical literature has highlighted the importance of accounting for within-industry firm heterogeneity to understanding the effects of international economic integration on average productivity and welfare (Melitz and Redding, 2013). A few recent papers, however, argue that firm heterogeneity is not important for welfare analysis. Arkolakis et al. (2012)

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argue that the welfare effect of trade is the same regardless of whether the firms are heterogeneous, as in Melitz (2003), with Pareto-distributed productivity or homogeneous, as in Krugman (1980), conditional on the import penetration ratio and trade elasticity. Similarly, Atkeson and Burstein (2010) show that trade costs can have a substantial impact on heterogeneous firms' exit, export, and process innovation decisions, but the impact of these changes on a country's welfare is largely offset in general equilibrium by product innovation (entry).

However, in order to focus on the reallocation effect, these studies analyze the effects of change in a trade cost in a setting with zero steady-state productivity growth. The extent to which the degree of firm heterogeneity impacts the long-run growth rate, and thereby welfare following trade and FDI liberalization, has received little attention.

More recently, a few papers have developed endogenous-growth models with a Melitz-type framework to examine the level and growth effects of trade liberalization (e.g., Baldwin and Robert-Nicoud (2008); Gustafsson and Segerstrom (2010); Unel (2010); Dinopoulos and Unel (2011) and Haruyama and Zhao (2008)), but they all restrict their analysis to a single mode of foreign-market entry, i.e. via exporting. There is much empirical evidence to suggest instead that firms frequently choose to serve foreign markets through FDI rather than exporting (Antras and R.Yeaple, 2013). Ample empirical evidence also tells us that the amount of knowledge spillovers from local affiliates of foreign multinationals is higher than from international trade (Keller, 2009). In light of this evidence, the omission of the activity of multinational firms in analyzing the long-run growth effect of openness is not negligible. With the possibility of serving the foreign market through horizontal-FDI, the way in which economic openness influences long-run growth and welfare in the presence of firm heterogeneity is an open question.

This paper attempts to fill this gap by introducing R&D spillovers in the discovery of new variety into the static trade and FDI heterogeneous firm model of Helpman et al. (2004). Specifically, we extend Helpman et al. framework into a variety-expansion endogenous-growth model in the tradition of Romer (1990). Alternatively, by allowing FDI as an additional mode of foreign-market entry, our model can also be viewed as an extension of the trade and endogenous- growth models with heterogeneous firms.

The R&D process uses labor and the available stock of technology. The model features externalities between firms that affect the *fixed costs* of innovation (including fixed costs of entry, production, export and FDI), and this generates endogenous growth. To capture the effect of foreign inventions on domestic R&D productivity, we allow for international trade and FDI as important channels of international technology spillover, which is endogenously determined in the model.

Innovation-based endogenous growth models usually lead to *scale effects*.[1] We employ a flexible R&D technology that allows us to introduce population growth and generates endogenous growth without generating strong *scale effects*. Specifically, we use two approaches to remove scale effects. The first approach allows for decreasing returns to scale to the knowledge stock —the *"increasing-complexity"* argument as in Jones (1995a). Under this assumption, trade and FDI have no impact on long-run growth rates, but only on the transition path. In our special case, by imposing a negative relationship between R&D productivity and market size, as in Segerstrom and Dinopoulos (1999), we show that

[1] Early idea-based endogenous-growth models such as (e.g., Romer (1990); Grossman and Helpman (1991) and Aghion and Howitt (1992)) feature a strong scale effect in which the long-run growth rate of an economy increases with the scale of the economy, measured by the size of population. Jones (1995b), using post war time series evidence from advanced countries, convincingly refuted the scale-effect prediction.

trade and FDI can affect the long-run growth rate without scale effects.

Unlike the seminal endogenous-growth literature, where productivity of all intermediate firms is assumed symmetric, a new intermediate good is introduced into the economy *under uncertainty*, in which a firm performs costly R&D investment to develop a new intermediate good (enters an industry) without knowing the productivity of its technology. Instead, as in Melitz (2003), the firm knows the distribution from which it can draw its productivity. We closely follow the Helpman et al. (2004) framework to characterize this production process in the intermediate-goods sector.[2] The presence of firm heterogeneity, coupled with the fixed costs, delivers an endogenous partitioning of firms with three productivity thresholds: domestic, export and FDI.

The model delivers several predictions regarding the impact of trade and FDI on growth and welfare. First, the presence of FDI raises the exit productivity cutoff by strengthening the selection process and, consequently, leads to a higher average productivity, as in Helpman et al. (2004). But it can also slow down the introduction of new varieties along the transition path to a new steady state or the steady-state growth rate.[3] A lower growth rate could arise because multinational presence has two countervailing effects on firms' incentive to develop new varieties. On the one hand, it accelerates the growth rate since multinationals - which are more productive than exporters - allow exploitation of international invention. On the other hand, it leads to tougher selection, hence fewer firms will find it profitable to develop new varieties. In other words firms need a more favorable productivity incentive in order to justify entry into the industry. Given our specification of international technology spillovers, the first effect is not adequate to offset the second and hence the growth rate becomes lower in the presence of multinationals. The effect of multinational presence on overall welfare is, therefore, ambiguous. These results imply that the static steady-state trade and FDI models (e.g., Helpman et al. (2004) and Ramondo and Rodriguez-Clare (2013)) tend to overstate the welfare gains from horizontal FDI. Therefore it is important to include both the static and dynamic component in any model that attempts to analyze the overall welfare effects of trade and FDI.

We further compare the welfare gains from trade in our model and in the trade-only models (TOMs, henceforth). We find that both the static [4] Melitz and Redding (2013) as well as the equivalent dynamic (Baldwin and Robert-Nicoud, 2008, and others) trade-only models with heterogeneous firms may yield inaccurate gains from trade. We show that the gain from trade (as measured by the elasticity of average productivity with respect to a change in trade cost) estimated in our model is smaller than in static TOMs without multinational production (MP).[5] Compared to dynamic TOMs, the growth effect of trade (as measured by the elasticity of the steady-state number of varieties or growth rate with respect

[2] Since firms in the intermediate-goods sector are heterogeneous in productivity, and due to the fixed market-entry costs, they take different organizational forms: they may serve only the domestic market; or serve a foreign market, either by producing in the foreign country to save variable and fixed trade costs or by exporting to avoid the additional fixed costs associated with setting up a new affiliate - the so called "proximity-concentration trade-off", as in Markusen (1984); Brainard (1997); Helpman et al. (2004).

[3] The qualitative result is very similar, whether or not multinational presence has a temporary or permanent effect on the growth rate.

[4] Note that the term *static* refers to all models that abstract from spillovers and hence generate zero steady growth rate.

[5] Our result is consistent with Ramondo and Rodriguez-Clare (2013) who find a similar result by incorporating FDI in the Ricardian model of international trade by Eaton and Kortum (2002) but their model doesn't allow for spillovers between firms.

to change in trade cost) is ambiguous and depends on inter-temporal and international spillovers. In the presence of MP as an alternative mode of foreign-market entry, both international spillovers and the ex-ante fixed cost of entry are *weakly increasing* with a reduction of trade cost since trade partially replaces FDI. Thus the gains from foreign technology spillover and loss due to uncertainty following trade liberalization would be overestimated if MP were omitted. If technology spillovers are exogenous, omitting MP as an alternative mode of entry can overestimate the negative growth effects of trade liberalization. These results imply that while the static TOMs tend to overestimate the productivity gain from trade, the dynamic TOMs can amplify the negative growth effect of trade cost reduction. Hence omitting MP as an alternative mode of foreign-market entry can severely overestimate or underestimate the overall welfare effect of trade cost reduction, depending on the degree of firm heterogeneity, fixed costs of trade, and FDI and inter-temporal spillovers.

The model also highlights the importance of firm heterogeneity in estimating the welfare implications of trade and FDI. If we allow for technology spillovers that change the relative cost of entry, firm heterogeneity can lead to a new dynamic loss from trade and FDI liberalization. This is because both the static gains and dynamic losses are stronger when the productivity dispersion among firms is larger. This shows that the gain from trade in a heterogeneous-firm model is not always larger than in homogeneous-firm models. Finally, our result indicates that the decentralized equilibrium is sub-optimal in comparison with the social planning solution.

To assess the quantitative implications of the model, we calibrate the model to the US economy. Our calibration implies that the degree of firm heterogeneity in productivity and R&D duplication externalities plays a pivotal role in explaining the welfare implications of a change in trade and FDI costs. For a wide range of plausible parameters, we find that a decrease in trade and FDI costs increases aggregate welfare. The aggregate welfare gains from trade and FDI cost reduction increase with the degree of firm heterogeneity.

The literature that comes closest to ours in combining heterogeneous firms and R&D-based endogenous-growth models are (e.g.,Baldwin and Robert-Nicoud (2008); Gustafsson and Segerstrom (2010); Unel (2010); Dinopoulos and Unel (2011)). These models, however, rule out FDI as a possible firm structure, which is the focus of this paper. Additionally, our model is related to a literature that explores the importance of firm heterogeneity in explaining the aggregate welfare gains from trade (seeArkolakis et al. (2012); Atkeson and Burstein (2010); Melitz and Redding (2013)). These studies, however, abstract from a positive long-run growth effect, which is at the core of our paper. Moreover, they ignore the presence of multinational production (MP) in their welfare calculation. Finally, our paper is related to Ramondo and Rodriguez-Clare (2013); Irarrazabal et al. (2013) and Wu (2013), who provide a quantitative analytical framework for analyzing the welfare implications of trade and MP in the presence of firm heterogeneity.

The rest of the paper is organized as follows. Section (2) lays down the main body of the model and Section (3) characterizes the steady-state equilibrium. Section (4) characterizes the steady-state impact of openness to trade and MP in the framework of a scale-invariant endogenous-growth model. Section (5) solves the model by imposing an assumption that firms' productivity follows a Pareto distribution. Section (6) compares the social optimum with the decentralized equilibrium. Section (7) calibrates the model and discusses the quantitative implications. Section (8) concludes the paper.

2 The Model

In the model there are two symmetric countries: home and foreign, denoted by H and F , respectively. Because of the symmetry assumption, we focus only on country H. Labor is the only primary factor, which can be used for creation of knowledge or production of final goods. Each country consists of two sectors: a perfectly competitive non-traded final goods sector that uses a combination of labor and a continuum of differentiated intermediate goods in production; and an intermediate-goods sector with Dixit-Stiglitz monopolistic competition where firms have an R&D division.

2.1 Household decisions

Each household is modeled as a dynastic family whose size grows exponentially at exogenous population growth rate $n > 0$. Normalizing the initial number of family members to one, the population level at time t is $L_t = e^{nt}$. Labor is employed either in the final-goods sector, L_{Yt}, or in the creation of knowledge, L_{It}, and it is perfectly mobile between the two sectors within an economy (but it is immobile across borders) and is paid the common wage rate w_t per unit of labor supplied.

Consumers in both countries have identical preferences and the inter-temporal utility of the households is given by:

$$U = \int_0^\infty \left(\frac{c_t^{1-\theta} - 1}{1-\theta} \right) e^{-(\rho - n)t} dt \tag{1}$$

where c_t denotes per-capita consumption of the home final good at time t, $\rho > n$ is the subjective discount rate of time preference, and $\theta > 0$ is the inverse of the inter-temporal elasticity of substitution.

The household maximizes the infinite lifetime utility given in (1) subject to the inter-temporal budget constraint:

$$\dot{a}_t = (r_t - n)a_t + w_t - c_t \tag{2}$$

where a_t is per-capita asset, w_t and r_t denote the wage rate and real interest rate on assets, respectively.

The inter-temporal optimization of (1) subject to (2) yields the usual Euler equation (see E.1 for the details):

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\theta} (r_t - \rho) \tag{3}$$

which entails that per-capita consumption c grows over time if and only if the market interest rate r exceeds the subjective discount rate ρ.

2.2 The final-goods sector

The final-goods sector in both countries is perfectly competitive and final goods are non-tradable. As in Romer (1990) and others, the non-traded final output Y_t is produced out of labor L_{Yt} and a continuum of differentiated home and foreign intermediate capital goods $q^c(\phi)$ with linearly homogeneous production technology of the form:

$$Y_t = A L_{Yt}^{1-\alpha} \sum_{c=H,F} \int_{\Omega^c} q_t^c(\phi)^\alpha dG^c(\phi) \tag{4}$$

where Y is final output, L_Y is labor input in final-goods production, $q^c(\phi)$ denotes quantity of an intermediate variety ϕ from country c available in the home country [6], Ω^H denotes the mass of intermediate capital goods produced by domestic firms and Ω^F is the mass of imported intermediate goods and goods produced at home by foreign multinationals, α parameterizes an intermediate input share and A denotes non-R&D - driven (exogenous) technology. Let P_Y denote the final-goods price and $p^c(\phi)$ the price of intermediate goods $q^c(\phi)$. The final output is chosen as a numéraire, and hence the price of final goods is $P_{Yt} = 1$.

Diverging from the common practice in endogenous-growth literature, where all the intermediate goods enter the production function symmetrically, firms producing intermediate varieties are heterogeneous over their productivity, ϕ, which is drawn randomly from a given probability distribution G, along the lines of Melitz (2003) and Helpman et al. (2004).

For convenience, we define a composite of intermediate capital goods as $K_t = \left[\sum_{c=H,F} \int_{\Omega^c} q^c(\phi)^\alpha dG^c(\phi)\right]^{\frac{1}{\alpha}}$ and hence output reads:

$$Y_t = A L_{Yt}^{1-\alpha} K_t^\alpha \tag{5}$$

In each country, the market for the final good is assumed to be perfectly competitive. Final good producers maximize (5) minus the cost of labor and intermediate capital used. The producer problem yields:

$$w_t = (1-\alpha) Y_t / L_{Yt} \text{ and } K_t P_t = \alpha Y_t \tag{6}$$

P_t denotes the aggregate price index associated with the intermediate sector at home and takes the following form:

$$P_t = \left[\sum_{c=H,F} \int_{\Omega^c} p^c(\phi)^{1-\sigma} dG^c(\phi)\right]^{\frac{1}{1-\sigma}} \tag{7}$$

where $\sigma = \frac{1}{1-\alpha} > 1$ is the elasticity of substitution between intermediate varieties. The perfect competition in the product market implies that $\left(\frac{w}{1-\alpha}\right)^{1-\alpha} \left(\frac{P}{\alpha}\right)^\alpha = P_Y$ and, given our choice of numéraire , this can be re-written as $\left(\frac{w}{1-\alpha}\right)^{1-\alpha} \left(\frac{P}{\alpha}\right)^\alpha = 1$; hence:

$$w_t = A^\sigma (1-\alpha) \alpha^{\sigma-1} P_t^{1-\sigma} \tag{8}$$

Equation (8) and the first equation of (6) gives:

$$Y_t = A^\sigma \alpha^{\sigma-1} P_t^{1-\sigma} L_{Yt} \tag{9}$$

[6] Each firm in the intermediate- goods sector produces a unique differentiated variety ω and, since each variety is linked to productivity, firms are indexed with a unique productivity level ϕ.

In the second stage, the firm decides on the optimal amount of input of each intermediate variety. Standard profit maximization gives the following demand function for intermediate goods (see E.2 for the derivation):

$$q_t^c(\phi) = \left[\frac{p_t^c(\phi)}{P_t}\right]^{-\sigma} \frac{\alpha Y_t}{P_t} \tag{10}$$

where αY_t is the aggregate expenditure on intermediate goods.

2.3 The intermediate-goods sector

Each firm in the intermediate-goods sector produces a unique differentiated variety. To keep the analysis simple, we assume that both innovation and production process are undertaken by the same firm. Unlike the seminal endogenous-growth models, a firm that enters an intermediate goods sector doesn't know the productivity of its product. Instead, along the lines of Melitz (2003), it knows the distribution from which its productivity draw will be realized. Hence different firms have different productivity levels.

2.3.1 Innovation

Firms must engage in costly R&D in order to generate knowledge capital using labor as an input. Without loss of generality, we assume that one unit of knowledge capital is produced using ζ_t units of labor at time t. The marginal cost of a unit of knowledge is then $w\zeta_t$. We assume that individual firms treat ζ_t as a parameter, but it can change over time due to a change in the stock of knowledge in domestic and foreign markets and the number of people looking for new ideas:[7]

$$\zeta_t = \frac{L_t^{\upsilon}}{(N_{dt} + \Upsilon N_{ft})^{\gamma} L_{It}^{\lambda-1}} \tag{11}$$

where N_{dt} and N_{ft} are the number of varieties produced in domestic and foreign markets, respectively, and L_I is the total labor in the economy devoted to innovation. The parameter γ captures the strength of inter-temporal knowledge spillover: if $\gamma > 0$, the productivity of labor increases with the stock of knowledge and this is often referred to as the "standing-on-the-shoulders effect"; $\gamma < 0$ represents the case where the arrival of new innovation declines as the stock of knowledge increases –the "fishing-out-effect". The parameter $\lambda \in [0, 1]$ denotes the negative externality of R&D duplication (0 if all innovations are duplication and 1 if there are no duplication innovations), this is the so-called "stepping-on-toes effect". The parameter $\upsilon > 0$ captures the "dilution effect"proposed by Segerstrom and Dinopoulos (1999), which reflects that R&D difficulty increases with the size of the market (measured by population size), i.e., all other things being equal, the larger υ, the bigger the decline in R&D labor productivity following an increase in population size. Finally, Υ captures international technology spillovers. We employ two different specifications of technology spillovers, Υ, through which foreign stock of knowledge impacts the productivity of domestic R&D labor.

Assuming symmetry, $N_{dt} = N_{ft}$, where N_{dt} is the number of varieties produced per economy, (11) can be re-written as:[8]

[7] As in Romer (1990) and others, we use the number of intermediate varieties (N_{dt}) as a proxy for the stock of knowledge.

[8] Note that the Helpman et al. (2004) framework abstracts from spillover between firms in the production of knowledge. By allowing such spillover, we demonstrate that the growth rate is endogenous.

$$\zeta_t = \frac{L_t^{v}}{(1+\Upsilon)^{\gamma} N_{dt}^{\gamma} L_{It}^{\lambda-1}} \tag{12}$$

There are four types of fixed costs in our model and these costs are interpreted as involving units of knowledge - they are denominated in units of R&D labor. A firm incurs a cost of $w\zeta f_e$ to develop a blueprint. In addition, it must pay $w\zeta f_d$ to adapt the product to the domestic market. A firm that decides to export must pay an additional fixed cost of $w\zeta f_x$ in order to export into the foreign market. If instead a firm decides to set up a foreign affiliate, it must pay $w\zeta f_m$ to establish a foreign subsidiary and serve the foreign market.[9]

2.3.2 Production

A firm innovates (enters an industry) under uncertainty, paying a fixed entry cost $w\zeta f_e$. Once a firms creates a blueprint (enters), it draws a productivity ϕ associated with its production from given productivity distribution $G(\phi)$. Depending on the productivity draw, the firm decides whether (and where) to produce and where to sell. The firm has the option of serving the foreign market by exporting or by opening a foreign affiliate. We closely follow the Helpman et al. (2004) framework, which incorporates firm heterogeneity in the traditional proximity-concentration trade-off, to characterize production in the intermediate-goods sector.

Production requires final goods as an input, and producing $q^c(\phi)$ of an intermediate good requires $\frac{q^c(\phi)}{\phi}$ units of the numéraire. A firm with productivity ϕ maximizes its "operating"profits (profit with out fixed costs) by choosing its optimal price p_j to serve the different markets, where $j \in \{d, x, m\}$ denotes domestic, export and multinational variables respectively:

$$\pi_{jt}(\phi) = \max_{p_{jt}(\phi)} \left[p_{jt}(\phi) q_{jt}(\phi) - \frac{q_{jt}(\phi)\tau^{\varsigma}}{\phi} \right] \tag{13}$$

subject to the demand function (10), where $\tau \in [1, \infty]$ is an iceberg cost. This optimization yields the standard result that equilibrium prices are a mark-up over marginal cost:

$$p_j(\phi) = \frac{\tau^{\varsigma}}{\alpha\phi} \tag{14}$$

where $\varsigma \in \{0, 1\}$ is an indicator variable of the export decision of firms (with $\varsigma = 1$ if the firm exports and zero otherwise). Note that p_j is inversely related to the firm's productivity ϕ: firms with higher productivity charge a lower price.

Multiplying the mark-up pricing rule (14) by the demand function (10), and substituting value of Y_t from (9), the revenue of a firm with productivity ϕ is given by:

$$A^{\sigma}\alpha^{2\sigma-1}\phi^{\sigma-1}\tau^{\varsigma(1-\sigma)}L_{Yt} \tag{15}$$

The operating profit of a firm is $(1-\alpha)$ times the revenue of the firm:[10]

$$\pi_{jt}(\phi) = A^{\sigma}(1-\alpha)\alpha^{2\sigma-1}\phi^{\sigma-1}\tau^{\varsigma(1-\sigma)}L_{Yt} \tag{16}$$

[9] As in the Helpman et al. (2004) framework, f_e, f_d, f_x , and f_m are fixed parameters.

[10] The operating profit $\pi(\phi) = p(\phi)q(\phi) - \tau^{\imath}q(\phi)/\phi \equiv q(\phi)p(\phi)[1 - \frac{\tau^{\imath}}{p(\phi)\phi}]$. Substituting for the value of $p(\phi)$ using (14), $\pi(\phi) = q(\phi)p(\phi)[1-\alpha]$.

This shows that operating profit is proportional to the productivity index $\phi^{\sigma-1}$ and the size of employment in final goods L_{Yt}. Since firms are symmetric except for their productivity draw, from (16), the profit of any two firms in the same country is as follows:

$$\frac{\pi_t(\phi)}{\pi_t(\phi')} = \left(\frac{\phi}{\phi'}\right)^{\sigma-1} \tag{17}$$

This indicates that the revenue and profits of firms in the same market depend only on their relative productivity.

Note that (16) represents the static profits of firms. We focus here on inter-temporal equilibrium, where the relevant element is the discounted present value of profits, $v_{jt}(\phi)$. The value of a firm in different markets is given by:

$$v_{jt}(\phi) = \int_t^\infty e^{-\int_t^\iota r(s)ds}\pi_{j\iota}(\phi)d\iota \tag{18}$$

where $-\int_t^\iota r(s)ds$ is the cumulative discount factor from time t to time ι.

Differentiating (18) with respect to time yields:

$$\pi_{jt}(\phi) + \dot{v}_{jt}(\phi) = r_t v_{jt}(\phi) \tag{19}$$

This equation represents the usual arbitrage condition. The LHS equals the return to equity in a firm with productivity level ϕ: owners of a firm earn the flow of profits $\pi_t(\phi)dt$ during the infinitesimal time interval dt and the capital gain $dv = \dot{v}_{jt}dt$. Solving for v_{jt} yields:

$$v_{jt}(\phi) = \frac{\pi_{jt}(\phi)}{r_t - [\dot{v}_{jt}(\phi)/v_{jt}(\phi)]}\ ,\ j = d,\ x{,}m \tag{20}$$

The discounted value of operating profits earned by a surviving firm equals the flow of profits π_{it} discounted by the market interest rate r_t minus the growth rate of v_{jt}.[11] Note that $\pi_{xt}(\phi)$ is defined as the profit from exporting only. If a firm sells in both export and domestic markets, then its aggregate profits amount to $\pi_{dt}(\phi) + \pi_{xt}(\phi)$ and corresponding aggregate discounted profits $v_{dt}(\phi) + v_{xt}(\phi)$. Similarly $\pi_{mt}(\phi)$ is defined as profit from producing abroad. The aggregate profit will be $\pi_{dt}(\phi) + \pi_{mt}(\phi)$ if a firm sells in the domestic market and serves the foreign market by establishing a subsidiary, with a corresponding discounted profit $v_{dt}(\phi) + v_{mt}(\phi)$.

To simplify the notation, the time subscript is henceforth omitted whenever this does not lead to confusion.

2.3.3 Productivity thresholds

We now determine the minimum levels of ϕ that firms choose to sell at home, export, and produce abroad. One can see from (15) that equilibrium profit is strictly increasing in ϕ, as does the discounted profit, i.e. $v_d(\phi)$, $v_x(\phi)$ and $v_m(\phi)$. Given the market entry-fixed costs, this implies that there are positive unique cutoff productivity levels:[12]

[11] If population growth rate is zero, i.e. $n = 0$, then $\dot{v}_{jt} = 0$ in the long run, hence (20) reduces to $v_{jt} = \pi_{jt}(\phi)/r_t$.

[12] Differentiating (21) with respect to time gives $\dot{v}_j/v_j = \dot{\zeta}/\zeta + \dot{w}/w$.

$$v_j(\phi) = w\zeta f_j \text{ where } j = d, x, m \tag{21}$$

A firm that has already incurred the fixed cost of developing a blueprint will enter a particular market if the discounted market-specific operating profit is positive. The productivity cutoff to enter the domestic market can be determined by finding the marginal firm that is indifferent between entering and not entering the local market. Substituting (16) and (20) into (21), the local market-entry condition is given by:

$$\frac{A^{\sigma}(1-\alpha)\alpha^{2\sigma-1}\phi_d^{*\sigma-1}L_Y}{r-(\dot{\zeta}/\zeta+\dot{w}/w)} = w\zeta f_d \tag{22}$$

Likewise, foreign-market entry via exporting is given by:

$$\frac{A^{\sigma}(1-\alpha)\alpha^{2\sigma-1}\phi_x^{*\sigma-1}hL_Y}{r-(\dot{\zeta}/\zeta+\dot{w}/w)} = w\zeta f_x \tag{23}$$

For convenience, the measure of iceberg cost, τ , for intermediates is transformed into $h = \tau^{1-\sigma}$ i.e $0 \leq h \leq 1$ ($h = 1$ costless trade and $h = 0$ when trade is perfectly closed).

Dividing (23) by (22) yields:

$$\phi_x^* = \epsilon\phi_d^* \text{ with } \epsilon = \left(\frac{f_x}{hf_d}\right)^{\frac{1}{\sigma-1}} \tag{24}$$

ϕ_x^* is an increasing function of the domestic productivity level, the variable trade cost and the ratio of overhead fixed costs f_x/f_d. As long as $\epsilon > 1$ (when $f_x \geq f_d$ and $h < 1$), the export productivity level is strictly greater than the domestic productivity level ϕ_d^*. Exporting is profitable for all firms with $\phi > \phi_x^*$ but very productive firms will find it optimal to serve the foreign market through FDI if the discounted profit from producing abroad exceeds the discounted profit from exporting. Solving the productivity level ϕ that yields the same discounted profit net of fixed costs between exports and FDI yields:[13]

$$v_m(\phi_m^*) - w\zeta f_m = v_x(\phi_m^*) - w\zeta f_x$$

$$\frac{A^{\sigma}(1-\alpha)\alpha^{2\sigma-1}\phi_m^{*\,\sigma-1}L_Y}{r-(\dot{\zeta}/\zeta+\dot{w}/w)} = \zeta w\frac{(f_m-f_x)}{(1-h)} \tag{25}$$

Dividing (25) by (23) yields the ratio of multinational to export cutoff:[14]

$$\phi_m^* = \ell\phi_x^* \text{ with } \ell = \left(\frac{(f_m-f_x)h}{f_x(1-h)}\right)^{\frac{1}{\sigma-1}} \tag{26}$$

ϕ_m^* is an increasing function of export productivity level ϕ_x^*, and the difference between the fixed cost of establishing a subsidiary and the fixed cost of export $(f_m - f_x)$ and h (or decreases with τ). For the model to be consistent with the empirical observation, we stipulate the productivity cutoff

[13] Because of the symmetry assumption, f_m is also incurred in units of home labor.

[14] Note that, since $r - (\dot{\zeta}/\zeta + \dot{w}/w)$ and ζ enter both the export and multinational cutoff, the conditions are identical to the static model of Helpman et al. (2004).

$\phi^*_m > \phi^*_x > \phi^*_d$. The condition for $\phi^*_m > \phi^*_x$, i.e. $\ell > 1$, reduces to $f_x/f_m < h < 1$. Hence, faced with a choice between exporting and FDI, more productive firms engage in the latter as long as $\ell > 1$.

Hence, given heterogeneity in productivity combined with fixed costs, the model generates four types of firms: non-producing firms; domestic firms d; exporting firms x; and multinational firms m. Firms that draw a productivity $\phi < \phi^*_d$ immediately exit the industry and will not be introduced. Firms with productivity $\phi^*_d \leq \phi < \phi^*_x$ serve only the local market. Firms with intermediate productivity $\phi^*_x \leq \phi \leq \phi^*_m$ serve the foreign market with exports. Firms that discover a variety with productivity level $\phi \geq \phi^*_m$ are able to become multinational. Fig.(**??**) presents this endogenous partitioning of firms according to their productivity draw.[15] We shall solve the model along the balanced growth path (BGP) where $\underline{\phi} < \phi^*_d < \phi^*_x < \phi^*_m < \bar{\phi}$ for all t.

In equilibrium, all firms with productivity level above ϕ^*_d are producing and the ex-post distribution of productivity conditional on successful market entry is given by:

$$\mu(\phi) = \begin{cases} \frac{g(\phi)}{1-G(\phi^*_d)} \text{ if } & \phi > \phi_d \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

2.3.4 Aggregation

We now present some aggregate variables of our model. We begin with the price index of the intermediate-goods sector (7), which is a CES aggregate as follows:[16]

$$\begin{aligned} P^{1-\sigma} \quad = \quad & \underbrace{\frac{1}{1-G(\phi^*_d)} \int_{\phi^*_d}^{\infty} p_d(\phi)^{1-\sigma} N_d g(\phi) d\phi}_{\text{Domestic firms}} + \underbrace{\frac{1}{G(\phi^*_m)-G(\phi^*_x)} \int_{\phi^*_x}^{\phi^*_m} p_x(\phi)^{1-\sigma} N_x g(\phi) d\phi}_{\text{Exporting firms}} \\ & + \underbrace{\frac{1}{1-G(\phi^*_m)} \int_{\phi^*_m}^{\infty} p_m(\phi)^{1-\sigma} N_m g(\phi) d\phi}_{\text{Foreign multinationals}} \end{aligned} \tag{28}$$

where $1 - G(\phi^*_d)$ is the ex-ante probability of successful firm entry. The probability of a foreign firm serving the domestic market via export or FDI is $1 - G(\phi^*_x)$ and the probability of serving only through FDI is $1 - G(\phi^*_m)$. Hence the probability of exporting only becomes $G(\phi^*_m) - G(\phi^*_x)$. N_d is the mass of domestically produced varieties by local firms, N_x is the number of foreign-produced varieties imported to the home market and N_m is the mass of domestically produced varieties by foreign multinationals. The total number of intermediate varieties available for production of a final good in any economy N is the sum of domestic firms (N_d), foreign exporters (N_x) and foreign multinationals (N_m) i.e. $N = N_d + N_x + N_m$.

The probability of foreign-produced varieties imported to the domestic market conditional on suc-

[15]The existence of such endogenous sorting of firms based on their productivity has been confirmed by firm-level empirical studies; see, for instance, Girma et al. (2004).

[16]Note that, because of the symmetry assumption, distribution of firms is identical across countries, i.e. $G^H(\phi) = G^F(\phi) = G(\phi)$. Note that $\mu(\phi)d\phi$ is the fraction of firms with productivity level ϕ, $N\mu(\phi)d\phi$ is the number of firms at that productivity level.

cessful entry is:

$$\eta_x \equiv \frac{N_x}{N_d} = \frac{G(\phi_m^*) - G(\phi_x^*)}{1 - G(\phi_d)} \tag{29a}$$

Likewise the probability of serving the domestic market through FDI conditional on successful market entry is given by:

$$\eta_m \equiv \frac{N_m}{N_d} = \frac{1 - G(\phi_m^*)}{1 - G(\phi_d^*)} \tag{29b}$$

The total number of intermediate varieties available for production of final good can be re-written as:

$$N = (1 + \eta_x + \eta_m)N_d \tag{29c}$$

Because of the assumption of symmetry, the probabilities (η_x and η_m) represent the fraction of all domestic firms that export and operate as multinationals.

After carrying out the calculation in A.1, the average-weighted productivity of all firms in each country is given by:

$$\tilde{\phi}^{\sigma-1} = \frac{N_d}{N}\left[\tilde{\phi}_d^{\sigma-1} + \eta_x \tau^{1-\sigma}\tilde{\phi}_x^{\sigma-1} + \eta_m \tilde{\phi}_m^{\sigma-1}\right] \tag{30}$$

where $\tilde{\phi}_d^{\sigma-1} = \frac{1}{1-G(\phi_d^*)}\int_{\phi_d^*}^{\infty} \phi^{\sigma-1} g(\phi) d\phi$ is the average weighted productivity of domestic firms, $\tilde{\phi}_x^{\sigma-1} = \frac{1}{G(\phi_m^*)-G(\phi_x^*)}\int_{\phi_x^*}^{\phi_m^*} \phi^{\sigma-1} g(\phi) d\phi$ and $\tilde{\phi}_m^{\sigma-1} = \frac{1}{1-G(\phi_m^*)}\int_{\phi_m^*}^{\infty} \phi^{\sigma-1} g(\phi) d\phi$ are the average productivities of foreign firms serving the domestic market through export and FDI, respectively.

Using this definition of weighted productivity, we can easily compute the aggregate variables of the model. The price index P_t can be written as functions of the mass of firms N_t and weighted-average productivity $\tilde{\phi}$ (see Appendix A.1 for the derivation)

$$P^{1-\sigma} = N(\alpha\tilde{\phi})^{\sigma-1} \tag{31}$$

Note that the price index for intermediates P falls as N increases.

One can easily obtain the (real) wage rate by plugging (31) into (8):

$$w = A^{\sigma}(1-\alpha)\alpha^{2(\sigma-1)}N\tilde{\phi}^{\sigma-1} \tag{32}$$

This indicates that the wage rate w increases with the total number of varieties available in the home market N.

Aggregate production of the final good can be obtained by substituting (32) in the first equation of (9).

$$Y = A^{\sigma}\alpha^{2(\sigma-1)}N\tilde{\phi}^{\sigma-1}L_Y \tag{33}$$

2.3.5 Free-entry condition

Having determined the aggregate variables and the cutoff facing a potential entrant, we now turn to the free-entry condition. Ex-ante firms are homogeneous and a firm's decision to create a new variety depends on the expected present value of developing a successful variety versus the expected fixed cost of entry.[17]

$$\begin{aligned}&\int_{\phi_d^*}^{\infty}(v_d(\phi)-w\zeta f_d)g(\phi)d\phi+\int_{\phi_x^*}^{\phi_m^*}(v_x(\phi)-w\zeta f_x)g(\phi)d\phi\\&+\int_{\phi_m^*}^{\infty}(v_m(\phi)-w\zeta f_m)g(\phi)d\phi=w\zeta f_e\end{aligned}\tag{34}$$

Using the cutoff conditions (22) (23) and (25), together with weighted-average productivities (30) and (32), the free-entry condition (34) can be expressed in the following compact form:[18]

$$\frac{\alpha L_Y}{N_d(r-(\dot{\zeta}/\zeta+\dot{w}/w))}=\zeta \mathrm{F}\tag{35a}$$

where,

$$\mathrm{F}=\frac{f_e}{1-G(\phi_d^*)}+f_d+\eta_x f_x+\eta_m f_m\tag{35b}$$

where $\frac{1}{(1-G(\phi_d^*))}$ is the probability of developing a successful variety. The expression on the LHS of (35a) captures the ex-ante discounted profits that a prospective entrant would obtain before observing its productivity draw from sales in both markets, and the RHS represents the ex-ante fixed cost of developing a successful variety. Hence the incentive to develop a new variety (or entry) is governed by the discounted net profit. F constitutes fixed costs and probabilities of foreign-market access conditional on successful market entry. Specifically, it captures a fixed cost of production in the domestic market f_d (which is a cost that every successful entrant incurs), the fixed cost of exporting f_x (which is faced only by a fraction of firms with probability η_x), the fixed cost of FDI (which is faced only by a fraction of firms with probability η_m) and the ex-ante product discovery cost of f_e multiplied by the inverse probability of successful market entry $1-G(\phi_d^*)$. Note that F is a very useful function that carries the effect of trade and MP on long-run productivity growth (we shall see this mechanism clearly in section 5.2.2.).[19]

Technological progress takes the form of introduction of new varieties in the intermediate-goods sector and the growth rate of innovation is dictated by the free-entry condition. It requires $(1/\mathrm{F}\zeta)$ units of labor to discover a successful new variety. Therefore the aggregate flow of new intermediate varieties produced in the economy is $\dot{N}_d=\frac{L_I}{\mathrm{F}\zeta}$, where $\dot{N}_d$ is the total number of new varieties developed in the economy and L_I is the aggregate quantity of labor devoted to R&D in the economy. By substituting the value of ζ from (12), the number of blueprints evolves according to:

[17] Note that the ex-ante likelihood of developing a successful variety ϕ is the same as the actual distribution of ϕ for successful firms already in the market.

[18] We assume that the financial sector is competitive and costless.

[19] Note that in the seminal trade and endogenous-growth literature, in which intermediate-goods firms are symmetric, there is no uncertainty in the productivity draw and F is an exogenous parameter.

$$\dot{N}_d = \frac{(1+\Upsilon)^{\gamma} N_d^{\gamma}}{\mathrm{F}} \frac{L_I^{\lambda}}{L^{\upsilon}} \tag{36}$$

Dividing both sides of (36) by N_d gives, in growth-rate form:[20]

$$g \equiv \frac{\dot{N}_d}{N_d} = \frac{L_I}{\mathrm{F}\zeta_t N_d} = \frac{(1+\Upsilon)^{\gamma} N_d^{\gamma-1} L_I^{\lambda}}{\mathrm{F} L^{\upsilon}} \tag{37}$$

A few remarks regarding the properties of the R&D equation are in order. Eq.(37) is a general knowledge production function and different assumptions on the parametric values produce different implications on the long-run growth effects of change in trade and FDI costs. In its simplest form, where $\gamma = 0$, $\upsilon = 0$ and $\lambda = 1$, the number of newly developed varieties $\dot{N}_d$ is proportional to the amount of labor devoted to R&D. Under $\gamma = 1$, $\upsilon = 0$ and $\lambda = 1$, this equation reduces to first-generation growth models as in Romer (1990); Baldwin and Robert-Nicoud (2008); Unel (2010) and others, which exhibits the strong form of scale effects. The scale-effect prediction, however, is convincingly refuted by empirical evidence Jones (1995b). One way of eliminating the scale effect is to assume $\upsilon = 1$, in which invention becomes increasingly difficult as the size of population increases, as in Segerstrom and Dinopoulos (1999); Dinopoulos and Unel (2011).[21] An alternative way to remove the scale effect is to allow decreasing returns to scale in the stock of knowledge, i.e. $\gamma < 1$ Jones (1995a); Segerstrom (1998); Gustafsson and Segerstrom (2010), which leads to a semi-endogenous rate of growth.[22] A crucial empirical debate now centers on the magnitude of the knowledge spillover parameter (γ), i.e how strongly the flow of new designs depends on the existing stock of knowledge. Most empirical studies lend strong support to the fully endogenous (FE) growth models Madsen (2008); Venturini (2012) and the semi-endogenous (SE) framework has also received support from Ang and Madsen (2011). Since the empirical test of semi-endogenous versus the second-generation endogenous-growth models is not conclusive, we keep our analysis as general as possible.

3 Steady-state Properties

We now solve the model for steady-state equilibrium in which all endogenous variables grow at a constant, but not necessarily the same, rate over time. First, along a balanced growth path (BGP), the growth rate of varieties, $g = \frac{\dot{N}_d}{N_d}$, must be constant over time. The free-entry condition (35a) implies that, since the growth rates of endogenous variables ($\tilde{\phi}$, N_d, L_Y and w) are all constant in the steady state, F must grow at a constant rate too. From (35b) it follows that η_x and η_m must be constant for F to grow at a constant rate. Hence the steady-state growth rate of imported varieties N_x , varieties produced by foreign multinational N_m and the total intermediate varieties available in the domestic market N are constant over time, i.e. $\frac{\dot{N}_x}{N_x} = \frac{\dot{N}_m}{N_m} = \frac{\dot{N}}{N} = g$.

[20]Note that Helpman et al. (2004) analyzes the effects of trade and MP in a setting where there is a zero steady-state growth, i.e. $g = 0$.

[21]Note that the intuition underlining this specification is similar to the Lucas-Uzawa specification Lucas (1988); Ziesemer (1991), where growth is driven by the share of human capital devoted to research activity or human capital per labor efficiency unit.

[22]See Jones (2005); Eicher and Turnovsky (1999) for a discussion on scale effect.

Steady-state cutoff conditions Equation (32) establishes that along a BGP, $\frac{\dot{w}}{w} = \frac{\dot{N}}{N} = g$, i.e. wage rate increases with the number of varieties and $\dot{\zeta}/\zeta = \upsilon\frac{\dot{L}}{L} - (\lambda-1)\frac{\dot{L}_I}{L_I} - \gamma\frac{\dot{N}_d}{N_d}$. In steady state, the growth in the number of R&D workers will be equal to the growth rate of the population, i.e. $\frac{\dot{L}}{L} = \frac{\dot{L}_I}{L_I} = n$, which is exogenous, and hence $\dot{\zeta}/\zeta = (\upsilon - \lambda + 1)n - \gamma g$. Using (32) to substitute for w, the local market-entry condition (22) in steady state can now be written as:

$$\frac{\alpha \phi_d^{*\sigma-1} L_Y}{r + (\gamma-1)g - (\upsilon - \lambda + 1)n} = N\tilde{\phi}^{\sigma-1}\zeta f_d \tag{38a}$$

where the LHS is associated with the discounted benefit of a firm having drawn $\phi = \phi_d^*$, while the corresponding RHS is associated with the costs of local entry. The profit from local sales is discounted by market interest rate r and the capital gain (loss) term $(\gamma-1)g - (\upsilon - \lambda + 1)n$.[23]

The corresponding entry condition via export is:

$$\frac{\alpha \phi_x^{*\sigma-1} L_Y h}{r + (\gamma-1)g + (\lambda - \upsilon - 1)n} = N\tilde{\phi}\zeta f_x \tag{38b}$$

Analogously, the entry condition for FDI is given as:

$$\frac{\alpha {\phi_m^*}^{\sigma-1} L_Y}{r + (\gamma-1)g + (\lambda - \upsilon - 1)n} = \frac{N\tilde{\phi}^{\sigma-1}\zeta(f_m - f_x)}{(1-h)} \tag{38c}$$

Likewise the free-entry condition can be re-written as:

$$\frac{\alpha L_Y}{r + (\gamma-1)g + (\lambda - \upsilon - 1)n} = N_d \zeta \mathrm{F} \tag{39}$$

Dividing (39) by (38a) and using (29c) yields ex-ante weighted cost of innovation:

$$\mathrm{F} = f_d \frac{\Phi}{\phi_d^{*\sigma-1}} \tag{40}$$

where $\Phi = \left(\frac{N}{N_d}\right)\tilde{\phi}^{\sigma-1} \equiv (1 + \eta_x + \eta_m)\tilde{\phi}^{\sigma-1}$

Solving the two equations of weighted cost of innovation (35b) and (40) together with the cutoff conditions yields (see Appendix A.2 for details of the derivation):

$$\begin{aligned} \frac{fe}{[1 - G(\phi_d^*)]} \quad = \quad & f_d\left[\left(\frac{\tilde{\phi}}{\phi_d^*}\right)^{\sigma-1} - 1\right] + \eta_x f_x \left[\frac{1}{h}\left(\frac{\tilde{\phi}}{\phi_x^*}\right)^{\sigma-1} - 1\right] + \\ & \eta_m f_m \left[\frac{f_m - f_x}{f_m(1-h)}\left(\frac{\tilde{\phi}}{\phi_m^*}\right)^{\sigma-1} - 1\right] \end{aligned} \tag{41}$$

[23]Note that, if population grows over time at a rate of n and $\gamma < 1$, there will be a capital loss only if $\frac{(\lambda-\upsilon-1)n}{1-\gamma} > g$, that is, when negative externalities of research duplication are sufficiently weak and/or crowding-out effect because of population growth is strong ($\lambda - \upsilon > 1$) and/or the positive externalities of knowledge spillovers are sufficiently strong (high γ). Note also that, if $\gamma = 1$, growth rate of new varieties becomes constant in steady-state if $\upsilon = \gamma$ and the denominator reduces to $r - n$.

Equilibrium in final-goods sector The final goods can be used for consumption or production of intermediate goods. The feasibility constraint on final goods is given by:

$$C + I = Y \tag{42}$$

where C is total consumption and I is the level of final output used in the production of intermediate goods, i.e. this can be interpreted as the economy's aggregate investment.

In order to produce an intermediate variety, a firm needs $q(\phi)/\phi$ units of the numéraire to produce $q(\phi)$, and this can be obtained by substituting (14) and (31) in (10) as follows:

$$\frac{q(\phi)}{\phi} = \frac{\tau^{-\imath\sigma}\alpha^2 Y}{N}\left(\frac{\phi}{\tilde{\phi}}\right)^{\sigma-1}$$

Summing over firms and using the definition of $\tilde{\phi}$ from (30), the aggregate demand for final goods by the intermediate-goods sector in steady state is expressed as:[24]

$$I = \alpha^2 Y = (\sigma - 1)\Pi = \alpha Y - \Pi \tag{43}$$

Equation (43) shows that the aggregate investment I is equal to aggregate expenditure on intermediate goods (αY) by the final-goods sector minus aggregate profits of the intermediate-goods sector Π.[25] From (42) and (43), aggregate consumption $C = (1 - \alpha^2)Y$. In (33) all variables are constant except N_d and L. This implies that, in steady-state equilibrium, aggregate production Y, aggregate investment I and aggregate consumption C grow over time at a rate of $g + n$, and hence individual consumer expenditure is growing over time at a rate of g. Hence, using the Euler equation (4), we can express market rate of return as $r = \theta g + \rho$.

Labor market equilibrium: The labor market is assumed to be perfectly competitive. The aggregate supply of labor is L, the aggregate demand for labor comes from fixed costs associated with different R&D activities (L_I) and production of final good (L_Y). Hence the full-employment condition reads:

$$L = L_I + L_Y \tag{44}$$

From (37), the labor devoted to R&D amounts to:

$$L_I = F\zeta g N_d \tag{45}$$

We use the free-entry condition (39) to solve for the steady-state value of L_Y

[24] To compute aggregate demand for final goods by the intermediate-goods sector, we integrate over firm demands for all active firms, $\frac{q(\phi)}{\phi}$.

$$\begin{aligned} I &= \tfrac{\alpha^2 Y}{\tilde{\phi}^{\sigma-1}N} \quad [\frac{1}{1-G(\phi_d^*)}\int_{\phi_d^*}^{\infty}\phi^{\sigma-1}N_d g(\phi)d\phi + \frac{h}{G(\phi_m^*)-G(\phi_x^*)}\int_{\phi_x^*}^{\phi_m^*}\phi^{\sigma-1}N_x g(\phi)d\phi \\ & \quad \frac{1}{1-G(\phi_m^*)}\int_{\phi_m^*}^{\infty}\phi^{\sigma-1}N_m g(\phi)d\phi] = \alpha^2 Y \end{aligned}$$

.

[25] Note that from $\Pi = N\pi(\tilde{\phi}) = \frac{\alpha Y}{\sigma}$.

$$L_Y = \frac{1}{\alpha} F\zeta N_d \left[r + (\gamma - 1)g + (\lambda - \upsilon - 1)n\right] \tag{46}$$

Substituting (45) and (46) into (44) yields:

$$F\zeta N_d \left\{\frac{r + (\alpha + \gamma - 1)g + (\lambda - \upsilon - 1)n}{\alpha}\right\} = L \tag{47}$$

Substituting the steady-state value of the interest rate and replacing for the value of ζ from (45) into (47), the share of labor devoted to innovation is:

$$\frac{L_I}{L} = \frac{1}{1 + \frac{1}{\alpha}\left[\frac{\rho + (\lambda - \upsilon - 1)n}{g} + \theta + \gamma - 1\right]} \tag{48}$$

Growth rate Finally, the growth rate can be obtained by inserting (48) back into (37) as:

$$g = \frac{1}{\theta + \alpha + \gamma - 1}\left[\alpha\left(\frac{g^{\lambda - 1}(1 + \Upsilon)^{\gamma} L^{\lambda - \upsilon}}{\mathrm{F} N_d^{1-\gamma}}\right)^{\frac{1}{\lambda}} - \rho - (\lambda - \upsilon - 1)n\right] \tag{49}$$

4 Steady-state Implications

We now use the model developed in the previous sub-section to characterize the steady-state impact of a change in trade and FDI costs by distinguishing two classes of scale-free models: first, we consider a model that features weak inter-temporal spillover, in the spirit of Jones (1995a); in the alternative specification, we assume that R&D difficulty rises with the size of population, as in Segerstrom and Dinopoulos (1999).

4.1 Semi-endogenous growth

Along the BGP, g , is constant by definition; the steady-state growth rate can easily be obtained by setting $\hat{g} = 0$ in (49):[26]

$$g^* = \frac{(\lambda - \upsilon)n}{1 - \gamma} \tag{50}$$

A positive steady-state growth $g^* > 0$ requires that both numerators and denominators have the same sign. Because population grows at an exogenous rate n and $\gamma < 1$, the steady-state growth rate will be positive if $\lambda > \upsilon$. According to (50), the steady-state rate of innovation increases with the rate of growth of research labor n; if the negative externality of R&D duplication with respect to research population $1 - \lambda$ is weaker, the positive knowledge spillovers effect of R&D γ is stronger and the lower the crowding-out effect (R&D difficulty) following an increase in population υ. This implies that, the rate of innovation along a BGP is determined merely by exogenously given variables.

Assumption 1. $\lambda > \upsilon$

[26]Note that when $\upsilon = 0$, this result coincides with the result obtained by Jones (1995a).

We impose this assumption because the empirically relevant cases for R&D-based economies are those featuring a positive economic growth rate.

Substituting (50) into (48) gives the steady-state share of labor in the R&D sector for the decentralized economy:

$$\mathtt{s}_{\mathtt{I}}^{*} = \frac{L_I}{L} = \frac{1}{1 + \frac{1}{\alpha}\left[\frac{(\rho-n)(1-\gamma)}{(\lambda-\upsilon)n} + \theta\right]} \tag{51}$$

Inspection of this equation reveals that, in a balanced growth situation, the fraction of labor allocated to R&D is invariant to trade and FDI costs but it is affected by several parameters in the model. An increase in the steady-state growth rate, $g^* = \frac{(\lambda-\upsilon)n}{1-\gamma}$, increases employment of labor in innovation. A lower rate of time preference ρ and higher inter-temporal elasticity of substitution $(1/\theta)$ is associated with a higher share of labor in the R&D sector. Finally, an increase in the share of intermediate goods in final output, α, increases the amount of labor for innovation because production of final output becomes less labor-intensive.[27]

We take per-capita real output as a measure of productivity.[28] From (31) and (32), the price index P for the intermediate sector falls along the BGP at a rate $g/(1-\sigma)$, while the wage rate increases at a rate of g, since N_d rises at the growth rate, g. By replacing L_Y from (51) into (33), the per-capita real output is written as:

$$y = \frac{Y}{L} = A^{\sigma}\alpha^{2(\sigma-1)}N_d\Phi(1-\mathtt{s}_{\mathtt{I}}) \tag{52}$$

Note that in the model's steady-state equilibrium, except for N_d, all other terms are constant over time; hence expansion of intermediate variety is the only source of productivity growth in the model and hence per-capita output grows at a rate of, g. Thus, if technological progress ceases, so will long-run per-capita growth. We obtain $C_t = (1-\alpha^2)Y_t$ and hence $c_t = (1-\alpha^2)y_t$, which also grows at a rate of g.

From (37) one can obtain the number of varieties N_d along the BGP:

$$N_d^* = \left\{L^{\lambda-\upsilon}\frac{s_I^{*\lambda}}{g^*}\left[\frac{(1+\Upsilon)^{\gamma}}{\mathrm{F}}\right]\right\}^{\frac{1}{1-\gamma}} \tag{53}$$

According to (53), there are two mechanisms through which trade and FDI costs affect the steady-state level of intermediate variety N_d^*: by changing the *ex-ante* fixed cost of developing a new variety F and/or flow of international knowledge Υ. The exact mechanism is deferred to (5.2.2), where explicit productivity distribution is assumed.

The steady-state level of per-capita output can be obtained by substituting (53) into (52):

$$y = A^{\sigma}\alpha^{2(\sigma-1)}\Phi\left\{L^{\lambda-\upsilon}\frac{s_I^{*\lambda}}{g^*}\left[\frac{(1+\Upsilon)^{\gamma}}{\mathrm{F}}\right]\right\}^{\frac{1}{1-\gamma}}(1-\mathtt{s}_{\mathtt{I}}^{*}) \tag{54}$$

[27]Note that if $n = 0$ or $\lambda = \upsilon$, the economy ends up in stagnation with all labor devoted to production of the final good.

[28]Since the final homogeneous output good is chosen as a numéraire, real output coincides with nominal production.

4.2 Endogenous growth

This sub-section examines the BGP with strong inter-temporal spillover $\gamma = 1$, which is a special case of the model above. This parametric restriction converts the model into a fully-endogenous growth model of the type proposed by Segerstrom and Dinopoulos (1999). To get a BGP under this case, we set the growth rate of g from (49) equal to zero; a constant growth rate of innovation along the BGP in the presence of positive population growth requires the knife-edge condition that $\lambda\frac{\dot{L}_I}{L_I} = \upsilon\frac{\dot{L}}{L}$ i.e. $\lambda = \upsilon$.

Assumption 2. $\lambda = \upsilon$

Imposing this assumption on (49) yields:[29]

$$g = \frac{1}{\theta + \alpha}\left[\alpha\left(\frac{g^{\lambda-1}(1+\Upsilon)}{\mathrm{F}}\right)^{\frac{1}{\lambda}} - (\rho - n)\right] \tag{55}$$

An important implication of this parametric restriction is that the long-run growth rate is now endogenous and captures different channels through which the seeds of openness to trade and MP impact the growth rate. Steady-state growth also depends on the effective discount rate $\rho - n$, inter-temporal elasticity of substitution $(1/\theta)$ and share of the intermediate goods sector α.

Note that, in the absence of population growth (i.e. $n = 0$), the integrated world economy experience exponential and endogenous long-run growth. In contrast, in the previous case where $\gamma < 1$ in (50), long-run growth vanishes if the population is not growing . However, this result requires the knife-edge condition that $\gamma = 1$ and $\lambda = \upsilon$, which is more restrictive than those conditions that generate semi-endogenous growth. Hence semi-endogenous is more general than endogenous growth.

5 Pareto Productivity Distribution

All the discussion up to now has been carried out without imposing a functional form on $G(\phi)$. In order to obtain explicit solutions for the cutoffs, steady-state level and growth rate, we now turn to solve the model by imposing an assumption that firm productivity follows a Pareto distribution.[30] The case of a Pareto productivity distribution has recently received particular attention in heterogeneous-firm trade literature for at least two reasons. First, empirical evidence Luttmer (2007); Helpman et al. (2004); Del Gatto et al. (2006) documents that the Pareto distribution approximates well the distribution of firm size in many countries. The Pareto distribution is also attractive for computational reasons. The CDF of the Pareto distribution is $G(\phi) = 1 - \left(\frac{\underline{\phi}}{\phi}\right)^k$ and the corresponding PDF is $g(\phi) = k\underline{\phi}^k\phi^{-(k+1)}$, where $\underline{\phi} > 0$ is the lower bound of the support of the productivity distribution; k corresponds to shape parameter and controls for the degree of dispersion.[31]

Assumption 3. *The productivity of firms in the intermediate-goods sector follows a Pareto dsitribution,* $G(\phi) = 1 - \left(\frac{\underline{\phi}}{\phi}\right)^k$ *, with the corresponding PDF,* $k\underline{\phi}^k\phi^{-(k+1)}$ *and* $\beta \equiv k/(\sigma - 1) > 1$.

[29] When $\gamma = 1$, the steady state does not exist unless $\lambda = \upsilon$ or $n = 0$.

[30] In particular, given the dependence of F and Υ on the conditional and unconditional distributions of the ϕ's, we cannot solve (54) and (55) explicitly without assuming an explicit functional form for G.

[31] The mean of the Pareto distribution $G(\phi) = 1 - \left(\frac{\underline{\phi}}{\phi}\right)^k$ is $\mathrm{m} = \frac{\underline{\phi}k}{k-1}$, while the variance is $\mathrm{v} = \frac{\mathrm{m}^2}{k(k-2)}$. A decrease in k increases both the mean and variance.

This assumption is plausible because it has received support from empirical studies. For instance, Helpman et al. (2004) find that sales distribution is Pareto with shape parameter $k-(\sigma-1)$ significantly greater than 0.

5.1 Steady-state cutoff productivity

With the assumption of a Pareto distribution, the weighted average productivity is (see Appendix A.3 for the derivation):

$$\Phi = \frac{\beta \phi_d^{*\sigma-1}}{\beta-1}\left[1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}}\right] \tag{56}$$

with Φ defined in (40). where, $\Lambda_{\mathrm{x}} = h^{\beta}\left(\frac{f_x}{f_d}\right)^{1-\beta}$, captures the protective effects of variable and fixed trade cost, $\Lambda_{\mathrm{x}}=0$ if $h=0$ ($\tau\to\infty$) and/or $\frac{f_x}{f_d}=\infty$ and $\Lambda_{\mathrm{x}}=1$ if $h=1$ ($\tau=1$) and $f_x=f_d$; $\Lambda_{\mathrm{m}} = \left(\frac{f_m-f_x}{f_d}\right)^{1-\beta}(1-h)^{\beta}$ measures the protective effects of FDI taking into account trade costs.

By inserting (56) into (40), it is straightforward to obtain:

$$F = f_d\left(\frac{\beta}{\beta-1}\right)\left[1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}}\right] \tag{57}$$

Alternatively, under a Pareto distribution (35b) can be written as (see A.4):

$$F = f_e\left(\frac{\phi_d^*}{\underline{\phi}}\right)^k + f_d\left(1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}}\right) \tag{58}$$

By equating (58) and (57), we obtain a closed-form solution for domestic productivity cutoff ϕ_d^* (see Appendix A.4 for details):

$$\phi_d^* = \underline{\phi}\left[\frac{f_d}{f_e(\beta-1)}\left\{1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}}\right\}\right]^{\frac{1}{k}} \tag{59a}$$

Note that when $(f_m-f_x)\to\infty$, the domestic cutoff condition collapses to that of the trade-only model as in Gustafsson and Segerstrom (2010). As long as the fixed cost establishing subsidiary f_m is not infinite and/or the variable trade cost is zero i.e. $h=1$, some firms always choose to serve the foreign market through FDI.

The steady-state cutoff conditions for exporting and serving with multinational are given by:

$$\phi_x^* = \underline{\phi}\left[\frac{f_x}{f_e(\beta-1)}\left\{1+\frac{1}{\Lambda_{\mathrm{x}}}\left[1+\Lambda_m\right]\right\}\right]^{\frac{1}{k}} \tag{59b}$$

and

$$\phi_m^* = \underline{\phi}\left[\frac{f_m-f_x}{f_e(\beta-1)}\left\{1+\frac{1}{\Lambda_{\mathrm{m}}}\left[1+\Lambda_x\right]\right\}\right]^{\frac{1}{k}} \tag{59c}$$

The weighted-average productivity can be obtained by substituting (59a) into (56):

$$\Phi = \beta\underline{\phi}^{\sigma-1}\left(\frac{f_d}{f_e}\right)^{\frac{1}{\beta}}\left[\frac{1}{\beta-1}\left(1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}}\right)\right]^{1+\frac{1}{\beta}} \tag{60}$$

For convenience, using (59a), (60) and (57) can be expressed in terms of the domestic productivity cutoff and parameters:

$$\Phi = \beta \frac{f_e}{f_d} \left(\frac{\phi_d^*}{\underline{\phi}} \right)^k \phi_d^{*\sigma-1} \tag{61}$$

$$\mathrm{F} = \beta f_e \left(\frac{\phi_d^*}{\underline{\phi}} \right)^k \tag{62}$$

In the next section we perform comparative statics to investigate the steady-state implications of a change in trade and FDI costs.

5.2 Effects of trade and FDI costs

As outlined above, the steady-state growth rate and share of R&D labor are unaffected by trade and FDI costs when $\gamma < 1$. Nevertheless, such a change in trade and FDI costs has a level effect on productivity and consumer welfare. To evaluate the level effect, one can easily examine what happens to (52), which combines two sources of productivity change, i.e. resource reallocation (captured by Φ) and variety effect N_d^*.

5.2.1 Reallocation effect

Let us start with the reallocation effect. Since firms are heterogeneous in the model, one source of productivity growth following a change in trade and MP costs could be through change in the composition of firms. In order to obtain better insight into how this works, we start by comparing the equilibrium for three states: closed economy (C) (defined as no trade and FDI), trade-only (T) and with both trade and FDI.

As the iceberg trade cost approaches infinity ($\tau \to \infty$ or $h \to 0$) and $(f_m - f_x) \to \infty$, the domestic cutoff productivity level approaches its closed-economy value $\phi_C^* = \underline{\phi} \left[\frac{f_d}{f_e(\beta-1)} \right]^{\frac{1}{k}}$. This implies that the autarkic cutoff productivity level ϕ_C^* is strictly less than the productivity cutoff in an open economy (59a). The domestic cutoff with in the absence of MP is $\phi_T^* = \underline{\phi} \left[\frac{f_d}{f_e(\beta-1)} \{1 + \Lambda_\mathrm{x}\} \right]^{\frac{1}{k}}$. Comparing the domestic cutoffs with and without MP indicates that the domestic cutoff increases in the presence of multinational activity.[32] This implies that the presence of multinational activity strengthens the selection process. From (61), weighted-average productivity increases monotonically in the domestic cutoff productivity level. Hence it is straight forward to show that the weighted-average productivity is higher compared to the trade-only models, except the limiting case of zero trade cost.

We now study the impact of further exposure to trade, a reduction in trade cost, and MP, a reduction in f_m, on productivity. To capture how productivity changes with changes in trade and investment cost, we clarify the effect on the cutoff productivity by differentiating (59a) and (59b) with respect to these costs. The following proposition summarizes this.

Proposition 1 (***Reallocation effect***)**.** *Given assumption (3), **(a)** Freer trade ($dh > 0$ and/or $df_x < 0$) leads to an increase in weighted-average productivity Φ as long as it retains the endogenous sorting of firms within the intermediate-goods sector, i.e. $\ell > 1$ or $f_x/f_m < h$. The productivity gain from trade cost reduction, however, is strictly lower than for trade-only models; **(b)** A reduction in FDI costs, $df_m < 0$, unambiguously increases the weighted-average productivity Φ.*

[32] It is clear to see that $\Lambda_\mathrm{m} > 0$, which indicates more openness rather than a measure of trade openness.

Proof See Appendix (C.1). $\square$

This result is similar to the results of Helpman et al. (2004) and Ramondo and Rodriguez-Clare (2013) but we derive this from a model with a positive steady-state productivity growth. The intuition of Proposition (1) is as follows. From (C.4) and (C.6) we can see that a reduction of iceberg cost (i.e. rises in h) has two effects. First, it raises the profits from exporting and thereby induces firms closest to the export productivity threshold prior to the change to start exporting, and this will increase competition since new firms are now using more resources to enter the export market, i.e. openness to trade increases $\frac{\partial \Lambda_x}{\partial h} > 0$. Second, it raises the profit from exporting relative to FDI and thereby makes it attractive to serve the foreign market through exports. This implies that least productive multinationals just to the right of the FDI cutoff prior to the change now switch to exporting (this is illustrated in Figure (1)). By switching to exporting, these firms will pay the trade costs but can avoid fixed costs of establishing a subsidiary. Openness to MP decreases $\frac{\partial \Lambda_m}{\partial h} < 0$. The question is: which effect dominates? The result shows that, as long as $\ell > 1$, i.e. $\frac{f_m}{f_x} > \frac{1}{h} > 1$, openness to trade dominates the decrease in MP openness and hence total openness increases, i.e. $\frac{\partial(\Lambda_\mathrm{x}+\Lambda_\mathrm{m})}{\partial h} >$, which leads to a tougher selection in the domestic market. However, $\frac{\partial(\phi_d^*/\phi_T)}{\partial h} < 0$, the selection effect from trade cost reduction is lower than in the pure trade case because it is partially offset by a decrease in the gain from FDI. This implies that the trade-only models, such as Melitz and Redding (2013), overestimate the effects of trade cost reduction on productivity due to the reallocation effect.

Figure 1 about here

A fall in FDI cost f_m enables the most productive exporters that previously shied away from FDI threshold to become multinational, which induces more competition since the new multinational firms are now using more $R\&D$ labor to cover the fixed costs of establishing a subsidiary compared to the trade costs. Moreover, less productive firms, which are just right of the original ϕ_x^* threshold prior to the change, stop exporting. This raises the domestic productivity cutoff, suggesting tougher selection and reallocation in the domestic market.

5.2.2 Variety effect

We now turn to look at the steady-state implication of a change in trade and FDI costs on the flow of variety, which is the second source of aggregate productivity. As noted before, policies could change the steady-state level of variety through the expected fixed cost of entry F (anti-variety) and/or international technology spillovers Υ (pro-variety). To explain the mechanics of this channel more fully, it is useful to explicitly specify Υ. Here we employ two specifications, Υ.

Case 1 (**Exogenous-technology spillover**).

Let us first consider the implication of the model in which technology is assumed to flow exogenously across countries, i.e. the spillover is unrelated to the degree of economic integration between countries. In this case, change in trade and FDI costs affects the steady-state level of variety only by changing the expected fixed cost of developing a new variety, F. F increases in response to a decrease in trade cost (τor f_x) as long as $\ell > 1$, which leads to a decrease in the rate at which new intermediate goods are produced.[33] But a decrease in the steady-state number of varieties due to a fall in trade cost is

[33] Noting that $\frac{\partial \phi_d^*}{\partial h} > 0$, $\frac{\partial \phi_d^*}{\partial f_x} < 0$ and $\frac{\partial \phi_d^*}{\partial f_m} < 0$; $\frac{\partial \mathrm{F}}{\partial h} > 0$ (or $\frac{\partial F}{\partial \tau} < 0$), $\frac{\partial \mathrm{F}}{\partial f_x} < 0$ and $\frac{\partial \mathrm{F}}{\partial f_m} < 0$.

smaller than in the trade-only model, as in Gustafsson and Segerstrom (2010), because an increase in F is partially offset by the exit of multinational firms.

$$\frac{\partial\left(\mathrm{F}/\mathrm{F_T}\right)}{\partial h} = -\frac{\ell^{\sigma-1-k} f_x \left(\mathrm{h}/\Lambda_\mathrm{x}+1\right)\beta}{\left(\mathrm{h}(\Lambda_\mathrm{x}+1)\right)^2} < 0 \tag{63}$$

Differentiating (53) after substituting F with respect to FDI fixed cost f_m, it is straight forward to show that the steady-state number of varieties decreases monotonically in response to a fall in f_m.

Case 2 (**Endogenous-technology spillover**).

The previous specification treated technology spillover as exogenous. Arguably spillover may not flow exogenously but depend on the degree of international economic linkages between countries. We employ an alternative specification in which technology spillover is a function of total sales of imports and sales of locally based foreign subsidiaries. Several studies have documented that technology spillover from horizontal FDI is higher than from imports (Keller, 2009). As already highlighted, due to fixed costs and firm heterogeneity, firms that opt for FDI are more productive than exporters (Tomiura, 2007), and this suggests another mechanism in which the potential spillover from horizontal FDI could be higher than learning from imported intermediate varieties.

With Pareto distribution, the spillover of international knowledge is given by(see Appendix (B)for derivation):

$$\Upsilon = \frac{\Lambda_\mathrm{x}+\Lambda_\mathrm{m}}{1+\Lambda_\mathrm{x}+\Lambda_\mathrm{m}} \tag{64a}$$

For convenience, we can re-write (64a) in terms of the domestic productivity cutoff and parameters:

$$\Upsilon = 1 - \frac{1}{(\beta-1)\frac{f_e}{f_d}\left(\frac{\phi_d^*}{\underline{\phi}}\right)^k} \tag{64b}$$

It is easy to see from (64b) that Υ increases in domestic productivity cutoff ϕ_d^*. Hence it increases in response to a reduction in trade costs (τ and f_x) for $f_x/f_m < h < 1$. But an increase in international knowledge spillover due to trade-cost reductions ($dh > 0$) is lower than in the case of the trade-only model because an increase in export sales is now partially offset by a decrease in sales of multinational firms. We show that both F and Υ increase in response to a reduction in trade and MP costs. The fundamental trade-off is now the following: does the effect on technology spillover exceed the increased expected cost of developing a new variety?

Substituting (62) and (64b) in (53) allows us to derive the steady-state level of varieties in terms of ϕ_d^* and parameters:

$$N_d^* = \left(L^{\lambda-\upsilon}\frac{s_I^{*\lambda}}{g^*}\right)^{\frac{1}{1-\gamma}} \left\{\frac{1}{(\phi_d^*/\underline{\phi})^k f_e\beta}\left[2-\frac{f_d}{(\phi_d^*/\underline{\phi})^k(\beta-1)f_e}\right]^\gamma\right\}^{\frac{1}{1-\gamma}} \tag{65}$$

Proposition 2 (***Variety effect***)**.** *When the return to knowledge is weak, in the spirit of Jones (1995a), in which $\gamma < 1$ and if assumptions (1) and (3) are satisfied:* ***(a)*** *Freer trade ($dh > 0$ and/or $df_x < 0$) reduces the steady-state level of domestic varieties as long as it preserves endogenous sorting of firms in the intermediate-goods sector, i.e. $\ell > 1$ or $f_x/f_m < h < 1$, but the loss in variety is lower in the endogenous Υ case compared to the benchmark exogenous Υ case;* ***(b)*** *With exogenous spillover, the* *elasticity of the steady-state number of varieties with respect to trade cost is lower than in the trade-only models; hence the cost of freer trade (in terms of variety loss) becomes smaller but this effect becomes ambiguous in the case of endogenous Υ; **(c)** As the degree of inter-temporal spillover becomes stronger, the elasticity of the steady-state number of varieties with respect to change in trade cost increases; strong γ amplifies the variety loss due to trade;**(d)** A reduction in FDI costs, $df_m < 0$, leads to a lower steady-state number of varieties.*

Proof See Apendix (C.2). □

The intuition underlying this result is as follows. Recall that the model captures two opposite forces that affect the productivity of labor in the innovation sector. First, a decrease in trade and MP costs increases the *ex-ante* expected cost of developing a new variety and hence firms need a more favorable productivity draw to justify entry into the domestic market. In other words, firms need to draw more on average from the Pareto distribution G in order to develop a profitable new variety (anti-variety). On the other hand, it increases the degree of international spillover, which reduces the cost of developing a successful variety (pro-variety). The question is whether the spillover effect is sufficiently strong to dominate the expected cost of entry. The latter effect is not present if the technology spillover is treated as exogenous and hence a reduction in trade costs unambiguously slows down the steady-state level of varieties. When the spillover is endogenous, although these effects works in opposite directions, it turns out that the anti-variety effect dominates the pro-variety effect. Strong inter-temporal spillover (high γ) amplifies the uncertainty more than the international spillover following a reduction in trade costs, and thus leads to a greater loss in variety.

5.2.3 Aggregate welfare implications

The results above highlight a trade-off between variety and productivity gain due to the reallocation effects of trade and FDI costs. While a reduction in trade and FDI costs improves the average weighted productivity by changing the composition of firms, it can also slow down the steady-state flow of new varieties. We now turn to examine the overall effects of greater economic integration on per-capita output.[34] To understand the impact of change in trade and FDI costs on steady-state per-capita output, as before we employ two specifications of technology spillovers.

By plugging (65) and (61) into (52), steady-state per-capita output can be written in-terms of ϕ_d^* and the model parameters:

$$y = \Delta \left(\frac{\phi_d^*}{\underline{\phi}}\right)^k \phi_d^{*\sigma-1} \left\{ \frac{1}{(\phi_d^*/\underline{\phi})^k f_e \beta} \left[2 - \frac{f_d}{(\phi_\mathrm{d}^*/\underline{\phi})^\mathrm{k} (\beta-1) \mathrm{f_e}} \right]^\gamma \right\}^{\frac{1}{1-\gamma}} \tag{66}$$

where $\Delta \equiv A^\sigma \alpha^{2(\sigma-1)} \beta \frac{f_e}{f_d} \left(L^{\lambda-\upsilon} \frac{s_I^{*\lambda}}{g^*} \right)^{\frac{1}{1-\gamma}} (1 - \mathtt{S_I})$, which is independent of trade and FDI costs.[35]

The following proposition summarizes the result:

Proposition 3 (***Overall welfare effect***)**.** *If $\gamma < 1$, in the spirit of Jones (1995a), assumptions (1) and (3) hold:*

[34] Alternatively, one can analyze the effect on real wage w to evaluate the welfare implications.

[35] In the case of exogenous spillover the terms in the square bracket become exogenous.

1. *With exogenous Υ, a reduction in trade cost ($dh > 0$ and/or $df_x < 0$) leads to a an increase in long-run per-capita output iff $\gamma < \frac{\sigma-1}{k+\sigma-1}$ as long as it preserves endogenous sorting of firms in the intermediate goods sector, i.e. $\ell > 1$ or $f_x/f_m < h < 1$. But the presence of MP reduces the elasticity of steady-state per-capita output with respect to change in trade cost; the TOMs could overestimate the welfare gain or welfare cost depending on the values of parameters γ, k and σ.*

2. *With endogenous $\Upsilon = \frac{\Lambda_\mathrm{x}+\Lambda_\mathrm{m}}{1+\Lambda_\mathrm{x}+\Lambda_\mathrm{m}}$, welfare improves with freer trade iff $\gamma < \frac{\sigma-1}{\sigma-1+k\left(1-\frac{1}{1+2\Lambda_x+2\Lambda_m}\right)}$ for $f_x/f_m < h < 1$ and with decrease in FDI cost iff $\gamma < \frac{\sigma-1}{\sigma-1+k\left(1-\frac{1}{1+2\Lambda_x+2\Lambda_m}\right)}$ for any value of f_m.*

Proof See Appendix (C.3). □

Proposition (3) highlights a complex interaction between openness and steady-state per-capita income when firms are heterogeneous. The result reveals that inter-temporal spillover γ and degree of firm heterogeneity k are crucial parameters that determine the sign and magnitude of the effect of change in trade and FDI costs. A fall in trade cost leads to an increase in per-capita output when inter-temporal spillover γ is relatively weak. Or, for a given inter-temporal spillover, the productivity distribution of firms must be large (low k) or the larger the elasticity of substitution between intermediate varieties combined (large σ). A higher inter-temporal spillover or low productivity dispersion implies that the productivity gain due to reallocation is dominated by the variety effect.

It is instructive to compare this result with the trade-only models as, in Gustafsson and Segerstrom (2010) that rule out FDI as foreign-market-entry mode. Although our model is similar to Gustafsson and Segerstrom (2010), the possibility of serving the foreign market with horizontal FDI makes the story of how trade policies impact growth in the presence of heterogeneous firms more complete. The trade-only model with heterogeneous firms predicts a monotonic increase in per-capita output with freer trade as long as inter-temporal spillover is weak. Our result points out that the no-FDI prediction on the sign of per-capita output holds if and only if $f_x/f_m < h < 1$. The welfare gain from trade in this paper can be higher or lower than the trade-only models, and this crucially depends on the degree of firm heterogeneity, inter-temporal and international spillovers.

5.2.4 Heterogeneity and aggregate welfare

As highlighted in the introduction, the extent to which (and whether) the existence of firm heterogeneity matters in estimating the aggregate welfare effects of trade has been the subject of recent controversy Arkolakis et al. (2012); Melitz and Redding (2013).[36] Hence it is worthwhile to investigate the effect of firm heterogeneity on the static and dynamic welfare effects of trade.

Since welfare essentially depends on the domestic productivity cutoff (ϕ_d^*), let us first consider the responsiveness of ϕ_d^* to a change in the degree of firm heterogeneity k. Differentiating ϕ_d^* with respect to k gives (see Appendix C.4 for the derivation):

$$\begin{aligned}\frac{d\ln(\phi_d^*/\underline{\phi})}{dk} &= -k^{-2}\ln(\phi_d^*/\underline{\phi})^k - \frac{k^{-1}}{(k-(\sigma-1))} \\ &\quad - \frac{k^{-1}\Lambda_x}{(1+\Lambda_\mathrm{x}+\Lambda_\mathrm{m})(\sigma-1)}\left\{\ln\epsilon^{\sigma-1}\left(1+\frac{\Lambda_\mathrm{m}}{\Lambda_\mathrm{x}}\ln\ell^{\sigma-1}\right)\right\} \equiv \xi_k < 0 \qquad (67)\end{aligned}$$

[36] Unlike our model, however, these studies abstract from growth effects and limit their analysis to trade.

In the absence of MP, this collapses to (similar to that in Melitz and Redding (2013)):

$$\frac{d\ln(\phi_d^*/\underline{\phi})}{dk} = -k^{-2}\ln(\phi_d^*/\underline{\phi})^k - \frac{k^{-1}}{(k-(\sigma-1))} - \frac{k^{-1}\Lambda_x \ln\epsilon^{\sigma-1}}{(1+\Lambda_x)(\sigma-1)} \equiv \xi_k^T < 0 \tag{68}$$

Comparison of (67) and (68) also shows that elasticity of domestic cutoff with respect to the degree of heterogeneity is higher in the presence of FDI than in the trade-only models as long as the most productive firms self-select into FDI, i.e. $\ell > 1$.

Proposition 4 (**Heterogeneity and aggregate welfare**). *Given assumption (3) and assuming endogenous selection of firms into domestic and foreign markets, i.e. $\epsilon > 1$ and $\ell > 1$, the greater the dispersion of firm productivity (smaller k): **(a)** leads to a larger welfare gain if and only if $k < (\sigma-1)\left(\frac{1}{\gamma}-1\right)$, **(b)** has an ambiguous effect on the welfare gain from a reduction in trade cost.*

Proof See Appendix (C.4). □

Proposition (4) establishes that the degree of firm heterogeneity matters in estimating the static and dynamic welfare effects of trade.[37] The result in Part (a) suggests a non-monotonic relationship between the degree of firm heterogeneity and aggregate welfare. To provide intuition, since a smaller k corresponds to greater productivity dispersion, ϕ_d^* increases with the level of firm heterogeneity, which from (61) implies the greater proportional static welfare gain due to a reallocation effect, i.e. higher Φ. An increase in firm heterogeneity (low k) exerts two competing effects on R&D productivity and then on dynamic welfare. On the one hand, as the right tail of the Pareto distribution becomes thicker (k decreases), more firms can potentially become multinational,international technology spillovers increases and hence R&D productivity increases. On the other hand, the *ex-ante* expected fixed cost of developing a new variety increases and thereby discourages the introduction of new intermediate varieties. For the exogenous Υ case, the first effect is absent and hence the steady-state number of varieties deceases as the "thickness"of the right tail of the productivity distribution increases. For endogenous Υ case,given our specification of technology spillover, the negative resource cost dominates the spillover and, as a result, the steady-state level of variety decreases as firms become more heterogeneous in productivity. The net effect of heterogeneity on aggregate welfare is, therefore, ambiguous. The key question would then be whether the variety effect dominates the reallocation effect. For exogenous Υ, greater productivity dispersion (lower k) implies larger y (welfare) if and only if $k < (\sigma-1)\left(\frac{1}{\gamma}-1\right)$. This implies, for a given inter-temporal spillover, that the degree of productivity dispersion must be substantially high, whereas the static models such as Melitz and Redding (2013) find that aggregate welfare improves monotonically with the degree of firm heterogeneity.

5.3 The special case: $\gamma = 1$

The following proposition describes the effects of a change in trade and FDI costs under the knife-edge assumption that inter-temporal spillover is unity, i.e. $\gamma = 1$.

Proposition 5 (**the case of** $\gamma = 1$). *When $\gamma = 1$, and if assumptions (3) and (2) are satisfied: **(a)** The economy grows along a BGP at a rate of:*

$$g = \frac{1}{\theta+\alpha}\left[\alpha\left\{\frac{g^{\lambda-1}}{(\phi_d^*/\underline{\phi})^k f_e \beta}\left[2 - \frac{f_d}{(\phi_d^*/\underline{\phi})^k(\beta-1)f_e}\right]\right\}^{\frac{1}{\lambda}} - (\rho-n)\right] \tag{69}$$

[37] We also estimate the effect of reduction in the FDI fixed costs, but we omit it for the sake of brevity.

(b) *The long-run growth rate g falls with freer trade as long as $f_x/f_m < h$, but the elasticity of growth with respect to trade cost reduction is lower than the no-FDI models. Openness to MP in the form of reduction in f_m unambiguously leads to a decrease in the steady-state growth rate.*

Proof See Appendix (C.5). □

The reasoning that drives this result is the same as in Proposition (2).[38]

6 Social Optimum

We now compare the decentralized equilibrium with the social optimum in order to identify the potential divergence due to various externalities. For the sake of exposition, in what follows we present only the main results: Appendix (D) provides the details. The optimum growth rate under social planning is:

$$g_s = \frac{1}{\gamma+\theta-1}\left\{g_s\left(\gamma+\left(\frac{1-S_I}{S_I}\right)\lambda\right)-(\rho+n(\lambda-\upsilon-1))\right\} \tag{70}$$

As in the market equilibrium, the optimal growth rate can be obtained by specifying two classes of models. When $\gamma < 1$, solving (70) gives the steady-state growth rate that is the same as (50) in decentralized equilibrium.

The socially optimal share of labor devoted to R&D is given by (see Appendix D):

$$\mathrm{S_I} = \frac{1}{1+\frac{1}{\lambda}\left[\frac{\rho(1-\gamma)}{(\lambda-\upsilon)n}+\theta-\gamma\right]} \tag{71}$$

Comparison of (71) and (51) shows the divergence between the optimal and the equilibrium solution. As discussed in Jones (1995a) the sources of divergence are the following. First, the fraction of labor devoted to R&D in decentralized equilibrium is lower than the social planner's solution when inter-temporal spillover is positive $\gamma > 0$ because private firms do not appropriate the value of the knowledge spillover that their discoveries impart. Second, the decentralized market allocates too much labor in R&D when there is negative externality due to R&D duplication i.e. $\lambda < 1$. Finally, the presence of the variety externality, α , in the final goods production (4) and (51) leads to underinvestment in R&D in the market economy.

The social optimum growth rate under the special case of $\gamma = 1$, and implementing the knife-edge condition $\lambda = \upsilon = 1$ is given by (see Appendix D):

$$g = \frac{1}{\theta}\left[\frac{1}{(\phi_d^*/\underline{\phi})^k f_e\beta}\left[2-\frac{f_d}{(\phi_d^*/\underline{\phi})^k(\beta-1)f_e}\right]-(\rho-n)\right] \tag{72}$$

When this solution is compared with the decentralized equilibrium in (69), the presence of variety externality in the final-goods sector (captured by α) in (4) and (69) makes steady-state growth in the decentralized solution sub-optimal.

[38] The same kind of qualitative behavior obtains for nearby values of γ, so the welfare and near-term growth effects of the model will be very similar whether or not the policy has permanent effects on the growth rate.

7 Calibration of the Model

In this section, we present some numerical examples to illustrate how growth and welfare respond to a change in trade and FDI costs along a BGP.

7.1 Parameterization

Table 1 reports the summary of the calibrated parameters.

Table 1: Calibrated Parameters

Parameter	**Value**	**Source**
Pareto distribution		
$\underline{\phi}$	1	Normalization
k	3.4	Ghironi and Melitz (2005)
Trade & FDI costs		
f_d	1	Normalization
τ	1.3	Ghironi and Melitz (2005)
f_e	5	
f_x	2	match 15% firm exporting
f_m	12.5	match 3% firm doing FDI
σ	3.8	Bernard et al. (2003)
The case $\gamma < 1$		
g	0.017	NIPA 7.1
λ	$\{0.5, 0.75, 1\}$	Jones and Williams (2000)
υ	0	Jones (1995a)
γ	$\{0.71, 0.56, 0.41\}$	implied
Special case $\gamma = 1$		
ρ	0.04	Garcia-Penalosa and Turnovsky (2005)
θ	2.5	Garcia-Penalosa and Turnovsky (2005)
$\lambda = \upsilon$	1	assumption
Both models		
n	0.01	NIPA 7.1

Following Ghironi and Melitz (2005), we set the elasticity of substitution across varieties $\sigma = 3.8$ (i.e. $\alpha = 0.74$), the productivity dispersion parameter $k = 3.4$, and the iceberg cost $\tau = 1.3$.[39] We assume the lower bound of the Pareto distribution to be 1. We also normalize f_d to 1. Given our choice for the parameters (k, σ,$\underline{\phi}$, f_d, τ), we choose f_e, f_x and f_m to guarantee that the model is consistent with the actual data. We choose $f_e = 5$, $f_x = 2$ and $f_m = 12.5$, consistent with the order of productivity cutoffs imposed in the model, namely $\phi_m^* > \phi_x^* > \phi_d^*$.[40] Our calibration implies that 66% of potential entrants

[39] The choice of σ and k is based on the estimates of plant-level US manufacturing data in Bernard et al. (2003), $\tau = 1.3$ is based on Obstfeld and Rogoff (2000).

[40] The minimum value of f_m that satisfies the ordering of productivity cutoffs given f_x and h is $f_m = 4.2$.

survive and engage in production for the domestic market; the probability of exporting conditional to domestic market entry is 15% ; while only 3% of firms are able to serve the foreign market with FDI.

We set the macroeconomic parameters to match the post 1970 growth experience of the US. Using NIPA Table 7.1, we set the average growth rate of per capita income g for the period 1970 - 2010 to 1.7% and a population growth rate of $n = 1\%$. Following Garcia-Penalosa and Turnovsky (2005) we set a coefficient of relative risk aversion of 2.5 and a rate of time preference of 4%.

7.2 Trade, FDI costs and welfare

To illustrate how growth and welfare respond to openness along a BGP, we start from the baseline calibration and change the iceberg trade cost h and fixed cost of FDI f_m. Figure 2 depicts the results of a change in variable trade cost from infinity, i.e. $\tau = \infty$ or $h = 0$, to zero variable trade cost, i.e. $\tau = 1$ or $h = 1$ (including the calibrated value of $h = 0.48$), on weighted-average productivity, steady-state variety, welfare gain and growth rate. The horizontal axis reports the transformed iceberg cost $h = \tau^{1-\sigma}$, which increases as τ decreases from ∞ to 1 since $\sigma > 1$.

Figure 2 about here.

Panel (A) of Figure 2 illustrates Proposition 1 by plotting the weighted-average productivity (with and without MP) versus the variable trade costs. In the absence of MP, weighted-average productivity is strictly increasing in h while in the presence of FDI it increases if $h > \frac{f_x}{f_m} = 0.16$, i.e if the most productive firms self-select into MP. It also illustrates that weighted-average productivity is higher in the presence of FDI than in the trade-only models, except in the limiting case in which variable trade cost is equal to zero, i.e. $h = 1$. The source of higher weighted-average productivity in FDI models is the presence of an additional adjustment margin due to FDI, which is absent in the TOMs.

In addition to the productivity and trade and FDI cost parameters, the calibration in Panels (B) and (C) requires more information about (λ, υ, γ and n). As Jones and Williams (2000) discussed, there is less agreement in empirical literature on the magnitude of these parameters. Following Jones (1995a) and others, we set $\upsilon = 0$. Using an R&D productivity equation, Jones and Williams (2000) estimated that the lower bound of $\lambda = 0.5$, and we present results for values of λ between 0.5 and 1. Given our choice for the parameters (υ, λ and n), γ was chosen (for different values of λ) to generate a steady-state growth rate of 1.7% in (50). For example, $\lambda \in \{0.5, 0.75, 1\}$ corresponds to $\gamma \in \{0.71, 0.56, 0.41\}$.[41] Panel (B) plots the result of Proposition 2 in which the steady-state number of varieties decreases in response to a decrease in variable trade costs. It also suggests that the presence of MP increases the dynamic welfare loss (due to loss in variety) when moving from a closed economy to an open economy.

Panel (C) depicts welfare gain from a move from a closed economy to an open economy with trade (y^T/y^C) and to trade and FDI (y/y^C) and variable trade cost for $\lambda = 0.75$ (this implies $\gamma = 0.56$).

Calibrating the model in Panel (D), which is a special case of our model, requires additional information on the values of θ and ρ. In order to obtain a BGP, we impose the knife-edge assumption that $\upsilon = \lambda = \gamma = 1$ and the parameters ρ, n and θ are calibrated according to Table 1. It illustrates that, in the absence of FDI, growth is strictly decreasing in h, while in the presence of FDI, trade cost reduction leads to lower growth if $h > \frac{f_x}{f_m}$. The result shows that the steady-state growth rate is strictly

[41] Note that, the larger the rate of population growth n, the smaller is γ.

lower than the growth in the TOMs, except in the limiting case of $h = 1$. The intuition concerning why the presence of FDI leads to a lower growth rate (steady-state number of varieties) is discussed in Proposition 2.

To illustrate how growth and welfare respond to a change in FDI fixed cost, we keep all the benchmark calibrated values and considers a reduction in fixed FDI costs from 30 to 5 (which includes our calibrated value of 12.5). The results are depicted in Figure 3.

Figure 3 about here.

7.3 The role of firm heterogeneity and R&D duplication externality

The degree of firm heterogeneity k and the R&D duplication externality, $1 - \lambda$, are critical parameters in our model. Hence, in the final set-up of our calibration we see the importance of these parameters in explaining the welfare effect of trade.[42] To see this, we keep all the benchmark calibrations, but change the value of k in Panel C-I and λ in Panel C-II of Figure 4.

Figure 4 about here.

Let us first consider the impact of the shape of the productivity distribution, k, on welfare. As discussed earlier, the shape parameter k affects static and dynamic welfare effect in opposite directions. Panel C-I of Figure 4 plots the relationship between welfare gain from openness (y/y^C) and variable trade cost under different parameterization of k. It illustrates that the welfare gains from trade and MP increase with the degree of firm heterogeneity (low k).

Similarly, Panel C-II of Figure 4 shows the relationship between welfare and trade cost under different parameterizations of λ. The degree of R&D duplication gives rise to two countervailing dynamic welfare effects. An increase in λ lowers the weight of the expected fixed cost of developing a new variety and international R&D spillover. Given our specification of the spillovers, the effect on the former channel is higher and hence dynamic welfare effects increase with λ.

Table 2 summarizes static and dynamic effect as we move from a closed economy ($h = 0$ and $f_m \to \infty$) to an equilibrium with only trade (y^T/y^C) ($f_m \to \infty$) and to the equilibrium with both trade and MP (WMP) (y/y^C). To provide intuition, we decompose the welfare effect into two parts: first, a static component that is identical to the trade and FDI models that abstract from growth effects, as Helpman et al. (2004); second, a dynamic term that accounts for a change in the steady-state number of varieties. Because of the dynamic welfare loss, indicated by a less than unity ratio of the dynamic components, the potential welfare gains from trade and FDI cost reduction are smaller than the static steady-state models, and this critically depends on the "stepping-on-toes"(λ) effect and the degree of firm heterogeneity (k). As noted before, the degree of heterogeneity has an ambiguous effect on welfare. But under the given parameter restriction, the static welfare effect dominates the dynamic negative effect. As a consequence, the welfare gain from trade and FDI increases as k decreases.

To sum up, our calibration suggests that aggregate welfare improves in response to a reduction in trade and FDI costs unless the degree of heterogeneity is relatively small (large k) and the magnitude

[42]While this sub-section focuses on a change in trade cost, a similar analysis can be undertaken for a change in fixed trade and FDI costs. For the sake of brevity, we omit this analysis.

Table 2: Calibration results: Welfare Cost of Autarky

	$\gamma < 1$				$\gamma = 1$			
	$k = 3.4$		$k = 4.25$		$k = 3.4$		$k = 4.25$	
	TOM	WMP	TOM	WMP	TOM	WMP	TOM	WMP
Static Component	1.74	2.43	1.41	1.62	1.74	2.43	1.41	1.62
Dynamic Component								
υ	0	0	0	0	1	1	1	1
$\lambda = 0.5$	0.62	0.42	0.75	0.64				
$\lambda = 0.75$	0.68	0.50	0.78	0.69				
$\lambda = 1$	0.70	0.55	0.79	0.71	0.91	0.81	0.96	0.93
Aggregate Welfare								
$\lambda = 0.5$	1.08	1.02	1.06	1.04				
$\lambda = 0.75$	1.18	1.22	1.10	1.12				
$\lambda = 1$	1.22	1.34	1.11	1.15	1.58	1.97	1.35	1.51

of duplication externalities is relatively large (low λ). When $k = 3.4$, the welfare gain from openness (with MP - WMP) is between 1.02 and 1.34, and this is on average higher than the welfare gain from openness computed under the trade-only models, which is between 1.08 and 1.22. The welfare gain from openness decreases when the magnitude of R&D duplication externalities (λ) is large. It is also interesting to observe that the welfare gain from MP decreases the larger the magnitude of R&D duplication externality (low λ).

Comparing the welfare implication of semi-endogenous and fully endogenous growth models, the welfare cost of autarky is lower in the case of fully endogenous growth models. But independently of the specification used, the welfare implication of a change in trade and FDI costs and degree of heterogeneity is qualitatively the same in both growth frameworks. The qualitative result obtains for near to 1 value of γ in the semi-endogenous case will be the same as the fully endogenous case. Hence the welfare and the near-term growth effects will be very similar whether or not changes in trade and FDI costs have a temporary or permanent effect on the growth rate.

8 Concluding Remarks

The static trade and FDI models with heterogeneous firms such as Helpman et al. (2004) suggest that openness to trade and FDI can increase productivity by stimulating reallocation of resources from less to more productive firms. By extending this static framework into a dynamic environment that features endogenous growth through R&D spillovers, we provide a rich set of predictions for the link between economic integration and long-run growth. We analytically show that the positive effect still holds, but it can be offset by a decrease in the steady-state flow of new intermediate variety. The aggregate welfare effects turn out to depend critically on the degree of firm heterogeneity and the magnitude of technology spillover parameters.

We present three important findings. First, we show that although multinational presence can improve average productivity through a selection process, it could also generate a dynamic loss by slowing down the introduction of new intermediate varieties. These results suggest that the static steady-state trade-FDI models tend to overstate the welfare gains from multinational presence.

Second, comparing this paper with the findings of the static and equivalent dynamic trade-only models demonstrates that introduction of FDI as an alternative mode of foreign market entry substantially affects the welfare implications of trade. Since trade substitutes for FDI, the gain from trade (as measured by the elasticity of average productivity of intermediate firms with respect to trade cost) is smaller than would be the case in the static TOMs, as in Melitz and Redding (2013). The dynamic welfare cost of trade (as measured by the elasticity of number of varieties or growth rate with respect to trade cost) is smaller than in the equivalent dynamic TOMs Baldwin and Robert-Nicoud (2008); Gustafsson and Segerstrom (2010). These results imply that the trade-only model may underestimate or overestimate the overall welfare effects of trade cost reduction.

Finally, it shows that firm heterogeneity plays an important role, affecting both the static and dynamic welfare effects of trade and MP. Unlike the static models, as in Melitz and Redding (2013), welfare gains from trade do not increase with degree of firm heterogeneity. Hence our model provides a unified framework that can be used to study the growth and welfare implications of trade and MP by taking a richer set of interactions.

Our calibration implies that firm heterogeneity and R&D duplication externality play a pivotal role in explaining the welfare gain from reduction of trade and FDI costs. For a wide range of parameters, we find that the degree of firm heterogeneity increases aggregate welfare from trade and MP. We also find that the presence of strong R&D duplication lowers the welfare gain from openness.

APPENDIX I

A Aggregate Outcomes

A.1 Weighted-average productivity

The CES price index from (7) can be written as:

$$
\begin{aligned}
P^{1-\sigma} &= \frac{1}{1-G(\phi_d^*)}\int_{\phi_d^*}^{\infty} p_d(\phi)^{1-\sigma}N_d g(\phi)d\phi + \frac{1}{G(\phi_m^*)-G(\phi_x^*)}\int_{\phi_x^*}^{\phi_m^*}(\tau p_d(\phi))^{1-\sigma}N_x g(\phi)d\phi \quad \text{(A.1)}\\
&+\frac{1}{1-G(\phi_m^*)}\int_{\phi_m^*}^{\infty} p_m(\phi)^{1-\sigma}N_m g(\phi)d\phi
\end{aligned}
$$

substituting for the mark-up price $p(\phi)$ from (14) in (A.1) yields:

$$
\begin{aligned}
P_t^{1-\sigma} &= \frac{1}{1-G(\phi_d^*)}\int_{\phi_d^*}^{\infty}(\alpha\phi)^{\sigma-1}N_d g(\phi)d\phi + \frac{\tau^{1-\sigma}}{G(\phi_m^*)-G(\phi_x^*)}\int_{\phi_x^*}^{\phi_m^*}(\alpha\phi)^{\sigma-1}N_x g(\phi)d\phi \quad \text{(A.2)}\\
&\frac{1}{1-G(\phi_m^*)}\int_{\phi_m^*}^{\phi_m^*}(\alpha\phi)^{\sigma-1}N_m g(\phi)d\phi
\end{aligned}
$$

replacing $\tilde{\phi}_d^{\sigma-1} = \frac{1}{1-G(\phi_d^*)}\int_{\phi_d^*}^{\infty}\phi^{\sigma-1}g(\phi)d\phi$, $\tilde{\phi}_x^{\sigma-1} = \frac{1}{G(\phi_m^*)-G(\phi_x^*)}\int_{\phi_x^*}^{\phi_m^*}\phi^{\sigma-1}g(\phi)d\phi$ and $\tilde{\phi}_m^{\sigma-1} = \frac{1}{1-G(\phi_m^*)}\int_{\phi_m^*}^{\infty}\phi^{\sigma-1}g(\phi)d\phi$ in A.2 gives:

$$P_t^{1-\sigma} = N_d(\alpha\tilde{\phi}_d)^{\sigma-1} + N_x\tau^{1-\sigma}(\alpha\tilde{\phi}_x)^{\sigma-1} + N_m(\alpha\tilde{\phi}_m)^{\sigma-1} \tag{A.3}$$

Average weighted productivity of all firms, $\tilde{\phi}^{\sigma-1}$, can be written as:

$$\tilde{\phi}^{\sigma-1} = \frac{1}{N_t}\left[N_d\tilde{\phi}_d^{\sigma-1} + N_x\tau^{1-\sigma}\tilde{\phi}_x^{\sigma-1} + N_m\tilde{\phi}_m^{\sigma-1}\right] \tag{A.4}$$

this can further be re-written as:

$$\tilde{\phi}^{\sigma-1} = \frac{N_d}{N_t}\left[\tilde{\phi}_d^{\sigma-1} + \eta_x\tau^{1-\sigma}\tilde{\phi}_x^{\sigma-1} + \eta_m\tilde{\phi}_m^{\sigma-1}\right] \tag{A.5}$$

substituting A.4 in A.3 yields 31 in the main text.

A.2 Steady-state cut-off equation (Equation (41))

In order to obtain the steady-state cutoff equation, we first equate (35b) and (40):

$$f_d\left(\frac{\tilde{\phi}}{\phi_d^*}\right)^{\sigma-1}\frac{N}{N_d} = \frac{f_e}{[1-G(\phi_d^*)]} + f_d + \eta_x f_x + \eta_m f_m \tag{A.6}$$

subtracting the RHS from the LHS

$$f_d\left[\left(\frac{\tilde{\phi}}{\phi_d^*}\right)^{\sigma-1} - 1\right] + \eta_x\left[f_d\left(\frac{\tilde{\phi}}{\phi_d^*}\right)^{\sigma-1} - f_x\right] + \eta_m\left[f_d\left(\frac{\tilde{\phi}}{\phi_d^*}\right)^{\sigma-1} - f_m\right] = \frac{f_e}{1-G(\phi_d^*)} \tag{A.7}$$

Substituting the cutoff conditions $\phi_m^* = \left(\frac{(f_m-f_x)h}{f_x(1-h)}\right)^{\frac{1}{\sigma-1}}\phi_x^*$ and $\phi_x^* = \left(\frac{f_x}{hf_d}\right)^{\frac{1}{\sigma-1}}\phi_d^*$ gives the cutoff equation (41) in the main text:

A.3 Productivity with Pareto distribution

This sub-section shows the derivation of aggregated productivity using a Pareto distribution. The CDF of the Pareto distribution

$$G(\phi) = 1 - \left(\frac{\underline{\phi}}{\phi}\right)^k \tag{A.8}$$

and the corresponding PDF

$$g(\phi) = k\underline{\phi}^k\phi^{-(k+1)} \tag{A.9}$$

Under Pareto distribution, the average weighted productivity of domestic firms is:

$$\tilde{\phi}_d^{\sigma-1} = \frac{1}{1-G(\phi_d^*)}\int_{\phi_d^*}^{\infty}\phi^{\sigma-1}g(\phi)d\phi \tag{A.10}$$

Substituting (A.8) and (A.9) in the above equation gives:

$$\tilde{\phi}_d^{\sigma-1} = \phi_d^{*k}k\int_{\phi_{d*}}^{\infty}\phi^{\sigma-k-2}d\phi = \phi_d^{*k}\frac{k}{\sigma-1-k}\left[\phi^{\sigma-k-1}\right]_{\phi_d^*}^{\infty} \tag{A.11}$$

Eq. (A.11) implies, for average productivity to have a finite mean, it is required that $k > \sigma - 1$. We define $\beta \equiv k/(\sigma - 1)$.

$$\tilde{\phi}_d^{\sigma-1} = \phi_d^{*\sigma-1}\left[\frac{k}{k-\sigma+1}\right] = \phi_d^{*\sigma-1}\left[\frac{\beta}{\beta-1}\right] \tag{A.12}$$

The average exporter productivity is:

$$\tilde{\phi}_x^{\sigma-1} = \frac{1}{G(\phi_m^*) - G(\phi_x^*)}\int_{\phi_x}^{\phi_m} \phi^{\sigma-1} g(\phi)d\phi$$

Using the CDF and PDF of the Pareto distribution:

$$\tilde{\phi}_x^{\sigma-1} = \frac{k}{\phi_x^{*-k} - \phi_m^{*-k}}\int_{\phi_x}^{\phi_m} \phi^{\sigma-k-2}d\phi = \frac{1}{\phi_x^{*-k} - \phi_m^{*-k}}\left(\frac{k}{\sigma-k-1}\right)\left[\phi^{\sigma-k-1}\right]_{\phi_x^*}^{\phi_m^*}$$

Using the definition of β

$$\tilde{\phi}_x^{\sigma-1} = \left(\frac{\beta}{\beta-1}\right)\frac{\left[\phi_x^{*\sigma-k-1} - \phi_m^{*\sigma-k-1}\right]}{\phi_x^{*-k} - \phi_m^{*-k}}$$

Using the cutoff definition $\phi_m^* = \ell\phi_x^*$, we get

$$\tilde{\phi}_x^{\sigma-1} = \frac{1-\ell^{\sigma-k-1}}{1-\ell^{-k}}\frac{\beta}{\beta-1}\phi_x^{*\sigma-1} \tag{A.13}$$

Note that average exporter productivity depends on: k, σ, f_x, f_m , τ (see (26) and the cutoff productivity ϕ_x^*. The average productivity of exporters increases with the export cutoff as long as $\ell > 1$.

Likewise, the average productivity of multinational firms is:

$$\tilde{\phi}_m^{\sigma-1} = \frac{1}{1-G(\phi_m^*)}\int_{\phi_m}^{\infty} \phi^{\sigma-1}g(\phi)d\phi = \phi_m^{*k}\frac{k}{\sigma-1-k}\left[\phi^{\sigma-k-1}\right]_{\phi_m^*}^{\infty}$$

$$\tilde{\phi}_m^{\sigma-1} = \frac{\beta}{\beta-1}\phi_m^{*\sigma-1} \tag{A.14}$$

This shows that the average multinational productivity depends on β (i.e. the elasticity of substitution σ and shape parameter k) and the cutoff productivity $\phi_m^{*\sigma-1}$.

The ratio of average productivity of exporter to multinational is:

$$\left(\frac{\tilde{\phi}_x}{\tilde{\phi}_m}\right)^{\sigma-1} = \frac{1-\ell^{\sigma-k-1}}{1-\ell^{-k}}\left(\frac{\phi_x^*}{\phi_m^*}\right)^{\sigma-1} = \frac{\ell^{1-\sigma+k}-1}{\ell^k-1} < 1 \tag{A.15}$$

Conditional on selling domestically, the probability of being an exporter is $\eta_x = \frac{G(\phi_m^*)-G(\phi_x^*)}{1-G(\phi_d^*)}$. Hence the fraction of firms that exports are η_x.

$$\eta_x = \frac{N_x}{N_d} = \frac{\phi_x^{*-k} - \phi_m^{*-k}}{\phi_d^{*-k}} = (1-\ell^{-k})\left(\frac{\phi_x^*}{\phi_d^*}\right)^{-k} \equiv (1-\ell^{-k})\epsilon^{-k} \tag{A.16}$$

The fraction of firms engaged in multinational production is:

$$\eta_m = \frac{N_m}{N_d} = \frac{1-G(\phi_m)}{1-G(\phi_d)} = \left(\frac{\phi_m^*}{\phi_d^*}\right)^{-k} \equiv (\epsilon\ell)^{-k} \tag{A.17}$$

Dividing (A.16) by (A.17) gives the relative mass of exporters to multinationals:

$$\frac{N_x}{N_m} = \frac{\eta_x}{\eta_m} = \frac{\phi_x^{*-k} - \phi_m^{*-k}}{\phi_m^{*-k}} = (1 - \ell^{-k}) \left(\frac{\phi_x^*}{\phi_m^*}\right)^{-k} \equiv \ell^k - 1 \tag{A.18}$$

This implies that as long as $\ell > 1$ and $k > 1$, an increase in FDI cost f_m makes exporting more attractive and lead to an increase in N_x/N_m. Given other things constant, the share of multinationals relative to exporters increase with the dispersion of the distribution (lower k).

Plugging (A.13) (A.14) (A.16) and (A.17) in (A.4) yields:

$$\frac{N}{N_d}\tilde{\phi}^{\sigma-1} = \frac{\beta}{\beta-1}\phi_d^{*\sigma-1}[1 + \tau^{1-\sigma}\epsilon^{\sigma-k-1} + \left(1 - \tau^{1-\sigma}\right)(\epsilon\ell)^{\sigma-k-1}] \tag{A.19}$$

Substituting the definition of the cutoff conditions ℓ and ϵ from (26) and (24) and using $h = \tau^{1-\sigma}$ in the above equation gives:

$$= \frac{\beta}{\beta-1}\phi_d^{*\sigma-1}\left\{1 + h\left(\left(\frac{f_x}{f_d h}\right)^{\frac{1}{\sigma-1}}\right)^{\sigma-k-1} + (1-h)\left(\left(\frac{f_m - f_x}{f_d(1-h)}\right)^{\frac{1}{\sigma-1}}\right)^{\sigma-k-1}\right\}$$

This is further simplified to:

$$= \frac{\beta\phi_d^{*\sigma-1}}{\beta-1}\left[1 + h^\beta\left(\frac{f_x}{f_d}\right)^{1-\beta} + \left(\frac{f_m - f_x}{f_d}\right)^{1-\beta}(1-h)^\beta\right] \tag{A.20}$$

Let $\Lambda_\mathrm{x} = h^\beta\left(\frac{f_x}{f_d}\right)^{1-\beta}$ and $\Lambda_\mathrm{m} = \left(\frac{f_m - f_x}{f_d}\right)^{1-\beta}(1-h)^\beta$

$$\Phi \equiv \frac{N}{N_d}\tilde{\phi}^{\sigma-1} = \frac{\beta\phi_d^{*\sigma-1}}{\beta-1}\left[1 + \Lambda_\mathrm{x} + \Lambda_\mathrm{m}\right] \tag{A.21}$$

A.4 Cutoff productivity with Pareto distribution

A.4.1 Derivation of (58)

From (A.16) and (A.17) we have

$$\eta_x = (1 - \ell^{-k})\epsilon^{-k} \text{ and } \eta_m = (\epsilon\ell)^{-k}$$

Substituting this values in (35b), the expected fixed cost can be written as:

$$\mathrm{F} = f_e\left(\frac{\phi_d^*}{\underline{\phi}}\right)^k + f_d\left(1 + \frac{f_x}{f_d}(1 - \ell^{-k})\epsilon^{-k} + \frac{f_m}{f_d}(\epsilon\ell)^{-k}\right)$$

This is can be further simplified as

$$\mathrm{F} = f_e\left(\frac{\phi_d^*}{\underline{\phi}}\right)^k + f_d\left[1 + \frac{f_x}{f_d}\epsilon^{-k} + \left(\frac{f_m - f_x}{f_d}\right)(\epsilon\ell)^{-k}\right]$$

Replacing the value of ϵ and ℓ from (26) and (24) gives (59a) in the main text.

A.4.2 Steady-state cut-off productivity levels (Derivations of equations (59a) and (59b))

By replacing the value of F from (58) into (57), we can obtain the cutoff productivity level for selling in the local market in (59a).

Using the cutoff definitions, the steady-state cutoff productivity for exporting is:

$$\phi_x^k = \tau^k (f_x/f_d)^{k/\sigma-1} \phi_d^k = h^{-\beta}(f_x/f_d)^{\beta} \phi_d^k = h^{-\beta}(f_x/f_d)^{\beta-1} f_x/f_d \phi_d^k$$

Substituting the domestic cutoff productivity from (59a) yields the export cutoff productivity in (59b) in the main text.

The steady-state cutoff productivity of serving the foreign market with FDI is:

$$\phi_m^k = (\ell\phi_x)^k = \left(\frac{f_m - f_x}{f_x(\tau^{\sigma-1}-1)}\right)^{\beta} \phi_x^k = \left(\frac{f_m - f_x}{f_x(\tau^{\sigma-1}-)}\right)^{\beta-1} \frac{f_m - f_x}{f_x(\tau^{\sigma-1}-)} \phi_x^k$$

Replacing the export cutoff productivity from (59b) in the above equation yields in multinational productivity cutoff in (59b) in the main text.

B International trade-FDI driven spillovers

Intermediate goods flow internationally. This section provides the derivation of the measures of international technology spillover through international trade and horizontal-FDI. As noted before the revenue is $\frac{1}{1-\alpha} \equiv \sigma$ times higher than the operating profits of the firm:

The derivation of (64a) in the main text:

$$\Upsilon = \Upsilon(\frac{\Gamma_{\mathrm{xm}}}{\Gamma}) = \frac{N_x \mathtt{r}(\tilde{\phi}_x) + N_m \mathtt{r}(\tilde{\phi}_m)}{N \mathtt{r}(\tilde{\phi})} \tag{B.1}$$

Combining (A.13) and (A.16) together with the cutoff conditions, the sales of the exporting firm is:

$$\Gamma_{\mathrm{x}} = \mathtt{r}(\tilde{\phi}_x) N_x = \sigma L_Y \alpha^{\sigma} \tau^{1-\sigma} \tilde{\phi}_x^{\sigma-1} N_x \tag{B.2}$$

$$\Gamma_{\mathrm{x}} = \sigma L_Y \alpha^{\sigma} \frac{\beta}{\beta-1} (\phi_d^*)^k \tau^{1-\sigma} \left(1 - \ell^{\sigma-k-1}\right) \phi_x^{*\sigma-k-1} N_d$$

$$\Gamma_{\mathrm{x}} = \sigma L_Y \alpha^{\sigma} \frac{\beta}{\beta-1} \tau^{1-\sigma} \left(1 - \ell^{\sigma-k-1}\right) \epsilon^{\sigma-k-1} N_d \phi_d^{*\sigma-1} \tag{B.3}$$

Similarly combining (A.14) and (A.17), the variable profit of multinational producers are:

$$\Gamma_{\mathrm{m}} = \mathtt{r}(\tilde{\phi}_m) N_m = \sigma L_{Yt} \alpha^{\sigma} \frac{\beta}{\beta-1} (\phi_d^*)^k \phi_m^{*\sigma-k-1} N_d$$

$$\Gamma_{\mathrm{m}} = \sigma L_Y \alpha^{\sigma} \frac{\beta}{\beta-1} (\ell\epsilon)^{\sigma-k-1} N_d \phi_d^{\sigma-1} \tag{B.4}$$

The ratio of (B.3) to (B.4) yields:

$$\frac{\Gamma_{\mathrm{x}}}{\Gamma_{\mathrm{m}}} = h\left(\ell^{k-(\sigma-1)} - 1\right) = h\left\{\left(\frac{(f_m - f_x)h}{f_x(1-h)}\right)^{\beta-1} - 1\right\} \tag{B.5}$$

It is interesting to note that $\frac{\Gamma_{\mathrm{x}}}{\Gamma_{\mathrm{m}}}$ tends to fall as trade costs rise (iceberg cost τ and the fixed cost of exporting f_x) and tends to rise as the fixed cost of establishing subsidiary f_m rises, which highlights the proximity-concentration trade-off between exports and local affiliate sales.

The total sales through export and FDI is:

$$\Gamma_{xm} = N_x \mathtt{r}(\tilde{\phi}_x) + N_m \mathtt{r}(\tilde{\phi}_m)$$

After some substitutions

$$\Gamma_{xm} = \sigma L_Y \alpha^{\sigma} \frac{\beta}{\beta-1} N_d \phi_d^{*\sigma-1} [\tau^{1-\sigma} \epsilon^{\sigma-k-1} + \left(1 - \tau^{1-\sigma}\right)(\epsilon\ell)^{\sigma-k-1}] \tag{B.6}$$

The total sales of all firms in the domestic economy can be obtained by substituting in ()

$$\Gamma = \mathtt{r}(\tilde{\phi})N = \sigma L_Y \alpha^{\sigma} \tilde{\phi}^{\sigma-1}(1 + \eta_x + \eta_m) N_d$$

Substituting (A.2) in the above equation yields:

$$\Gamma = \sigma L_Y \alpha^{\sigma} \frac{\beta}{\beta-1} \phi_d^{*\sigma-1} N_d [1 + \tau^{1-\sigma} \epsilon^{\sigma-k-1} + \left(1 - \tau^{1-\sigma}\right)(\epsilon\ell)^{\sigma-k-1}] \tag{B.7}$$

The ratio of (A.21) to (B.7) gives the measure of international spillover:

$$\Upsilon = \frac{\Gamma_{\mathrm{xm}}}{\Gamma} = \frac{\tau^{1-\sigma}\epsilon^{\sigma-k-1} + \left(1 - \tau^{1-\sigma}\right)(\epsilon\ell)^{\sigma-k-1}}{1 + \tau^{1-\sigma}\epsilon^{\sigma-k-1} + \left(1 - \tau^{1-\sigma}\right)(\epsilon\ell)^{\sigma-k-1}} \tag{B.8}$$

Substituting the cutoffs conditions for ℓ and ϵ in (B.8) gives:

$$\Upsilon = \frac{\Gamma_{\mathrm{xm}}}{\Gamma} = \frac{h^{\beta}\left(\frac{f_x}{f_d}\right)^{1-\beta} + \left(\frac{f_m - f_x}{f_d}\right)^{1-\beta}(1-h)^{\beta}}{1 + h^{\beta}\left(\frac{f_x}{f_d}\right)^{1-\beta} + \left(\frac{f_m - f_x}{f_d}\right)^{1-\beta}(1-h)^{\beta}} \tag{B.9}$$

It is also worth noting that the distributional parameter k (combined with the elasticity of substitution σ), has an impact on the degree of knowledge spillover Υ. The greater the variance of firms productivity (small k) within the intermediate goods sector, the larger the degree of international technology spillover Υ as long as there is self-selection of most productive firms to serve foreign market with FDI, i.e. $f_x/f_m < h < 1$. This can be shown by changing the value of k, holding all other parameters of the model constant.

$$\frac{\partial \Upsilon}{\partial \mathrm{k}} = -\frac{\Lambda_x \ln \epsilon^{\sigma-1}}{(\sigma-1)(1 + \Lambda_x + \Lambda_m)}\left(1 + \frac{\Lambda_m}{\Lambda_x}\ln \ell^{\sigma-1}\right)[1 - \Upsilon] < 0 \tag{B.10}$$

To understand the intuition behind this result, it is interesting to see how the ratio of sales of exporters to multinationals change $\frac{\Gamma_{\mathrm{x}}}{\Gamma_{\mathrm{m}}}$ as a result of change in k. We can easily see from the ratio of sales of exporters to multinational $\frac{\Gamma_{\mathrm{x}}}{\Gamma_{\mathrm{m}}}$ with respect to h from B.5. A decrease in k raises the density of the function where firms find MP optimal relative to the the density where firms choose exporting. Since multinational firms are more productive than exporting firms, this will be associated with higher level of international knowledge spillover.

C Proofs of propositions

In this section we present the details of the propositions.

C.1 Proof of Proposition (1)

Proof First, we obtain:

$$\frac{\partial \Lambda_x}{\partial h} = \left(\frac{f_x}{hf_d}\right)^{1-\beta} \beta = \epsilon^{\sigma-1-k}\beta > 0 \text{ and } \tfrac{\partial \Lambda_m}{\partial h} = -\left(\tfrac{f_m - f_x}{f_d(1-h)}\right)^{1-\beta} \beta = -(\ell\epsilon)^{\sigma-1-k}\beta < 0$$

$$\frac{\partial(\Lambda_{\mathrm{x}} + \Lambda_{\mathrm{m}})}{\partial h} = \left(\frac{f_x}{hf_d}\right)^{1-\beta} \beta \left(1 - \left(\frac{(f_m - f_x)h}{f_x(1-h)}\right)^{1-\beta}\right) = \epsilon^{\sigma-1-k}\beta\left(1 - \ell^{\sigma-1-k}\right) > 0$$

The elasticity of ϕ_d^* and Φ can be obtained by differentiating (59a) and 60) with respect to h:

$$\frac{\partial \phi_d^*}{\partial h}\frac{h}{\phi_d^*} = \frac{\Lambda_{\mathrm{x}}(1 - \ell^{\sigma-1-k})}{(\sigma-1)(1 + \Lambda_{\mathrm{x}} + \Lambda_{\mathrm{m}})} \equiv \xi_d > 0 \text{ for } \ell > 1 \tag{C.1}$$

$$\frac{\partial \Phi}{\partial h}\frac{h}{\Phi} = \beta\frac{\Lambda_{\mathrm{x}}(1 - \ell^{\sigma-1-k})}{(1 + \Lambda_{\mathrm{x}} + \Lambda_{\mathrm{m}})}(1 + \frac{1}{\beta}) = (k + \sigma - 1)\xi_d$$

Similarly, the elasticity of ϕ_d^* with respect to fixed export cost f_x is:

$$\frac{\partial \phi_d^*}{\partial f_x}\frac{f_x}{\phi_d^*} = -\frac{(k - (\sigma-1))}{k(\sigma-1)}\frac{\Lambda_{\mathrm{x}}(1 - \ell^{-k})}{(1 + \Lambda_{\mathrm{x}} + \Lambda_{\mathrm{m}})} < 0 \text{ for } \ell > 1$$

$$\frac{\partial \Phi}{\partial f_x}\frac{f_x}{\Phi} = (k + \sigma - 1)\frac{\partial \phi_d^*}{\partial f_x}\frac{f_x}{\phi_d^*} < 0 \text{ for } \ell > 1$$

The elasticity of domestic cutoff with respect to trade costs is an endogenous variable, which varies with the level of trade costs, productivity dispersion and endogenous sorting of firms into domestic and foreign market. The behaviour of the domestic cutoff will entirely determine the welfare effect. The domestic cutoff and weighted average productivity rises with reduction of variable (higher h) and fixed (lower f_x) trade costs as long as $\ell > 1$. But it the magnitude of elasticity with respect to variable and fixed costs are different.

In the absence of multinational ($f_m \to \infty$), the elasticity of domestic cutoff to change in h [43]

$$\frac{\partial \phi_T^*}{\partial h}\frac{h}{\phi_T^*} = \frac{\Lambda_{\mathrm{x}}}{(\sigma-1)(1 + \Lambda_{\mathrm{x}})} > 0 \tag{C.2}$$

Comparing (C.2) with no-FDI model (C.1), as in Melitz and Redding (2013), the presence of MP give rise to a lower elasticity of domestic cutoff productivity with respect to a reduction in variable trade cost (high h), which implies lower static gain from reduction in trade cost due to reallocation effect. Alternatively:

$$\frac{\partial \ln(\phi_d^*/\phi_T^*)}{\partial h} = \xi_d - \xi_d^T < 0 \tag{C.3}$$

[43]Note that this is the same as Melitz and Redding (2013).

The elasticity of export cutoff with respect to h is:

$$\frac{\partial\phi_x^*}{\partial h}\frac{h}{\phi_x^*} = -\frac{1}{(\sigma-1)}\frac{(1+\Lambda_{\mathrm{m}}/(1-h))}{(1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}})} < 0 \tag{C.4}$$

In the absence of MP, the elasticity of export cutoff with respect to reduction in trade cost is:

$$\frac{\partial\phi_x^*}{\partial h}\frac{h}{\phi_x^*} = -\frac{1}{(\sigma-1)}\frac{1}{1+\Lambda_{\mathrm{x}}} < 0 \tag{C.5}$$

Similarly the elasticity of MP cutoff with respect to h is given by:

$$\frac{\partial\phi_m^*}{\partial h}\frac{h}{\phi_m^*} = \frac{1}{(\sigma-1)}\frac{(h/1-h)\,(1+\Lambda_{\mathrm{x}}/h)}{(1+\Lambda_{\mathrm{x}}+\Lambda_m)} > 0 \tag{C.6}$$

The effect of change in FDI cost can be obtained through partial differentiation of (59a) and (59b) with respect to f_m .

$$\frac{\partial\phi_d^*}{\partial f_m}\frac{f_m}{\phi_d^*} = -\frac{(k-(\sigma-1))}{k(\sigma-1)}\left(\frac{\Lambda_{\mathrm{m}}}{1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}}}\right) < 0 \text{ for } \beta > 1 \tag{C.7}$$

□

C.2 Proof of Proposition (2)

Proof (a) First, consider the case where international technology spillover is exogenous. In this case, the elasticity of steady-state number of varieties with respect to h is given by:[44]

$$\frac{dN_d^*}{dh}\frac{h}{N_d^*}dh = -\frac{k}{1-\gamma}\xi_d < 0 \tag{C.8}$$

In the case of endogenous spillover, differentiating (53) after substituting for (62) and (64b) yields:

$$\frac{dN_d^*}{dh}\frac{h}{N_d^*}dh = -\frac{k}{1-\gamma}\xi_d\left[1-\frac{\gamma}{1+2\Lambda_{\mathrm{m}}+2\Lambda_{\mathrm{x}}}\right]dh \tag{C.9}$$

Note that a decrease in variable trade cost lead to an increase long-run level of varieties only if the term in square bracket is greater than zero i.e. $\gamma > 1+2\Lambda_{\mathrm{m}}+2\Lambda_{\mathrm{x}} \equiv \vee$ or $\ell < 1$. But this formally violates our restriction $\gamma < 1$ and $\ell > 1$. Hence as long as $\gamma < 1$ and $\ell > 1$, a reduction in trade cost lead to a lower steady-state level of varieties. This implies that, with the assumed functional form, knowledge spillover is not sufficient to dominate the resource cost which is generated by higher expected cost of developing a new variety.

For example, if $\gamma = 0$, then $\frac{dN_d^*}{dh}\frac{h}{N_d^*} = -\beta(1-\ell^{\sigma-1-k})\left(\frac{\Lambda_x}{1+\Lambda_{\mathrm{m}}+\Lambda_{\mathrm{x}}}\right) = -k\xi_d$.

Comparing (C.12) and (C.9) suggest that, for $\gamma \in (0,1)$, the elasticity of steady-state variety with respect to change in trade cost is higher in the case of exogenous international spillover.

(b) In the absence of FDI, the elasticity of steady-state number of variety with respect to change in trade costs in the case of exogenous Υ is given by:

$$\frac{dN_d^*}{dh}\frac{h}{N_d^*}dh = -\frac{k}{1-\gamma}\xi_d^T < 0 \tag{C.10}$$

[44] The results will be the same (except the magnitude of the elasticity) if one considers reduction in fixed cost of export f_x rather than an increase in h. For the sake of brevity, we skip this trivial exercise.

and in the case of endogenous Υ

$$\frac{dN_d^*}{dh}\frac{h}{N_d^*}dh = -\frac{k}{1-\gamma}\xi_d^T\left[1-\frac{\gamma}{1+2\Lambda_{\mathrm{x}}}\right]dh \tag{C.11}$$

(c) The degree of inter-temporal spillover is cretical in determining the dynamic effect. Differentiating (C.9) with respect to γ gives:

$$\frac{d}{d\gamma}\left(\frac{dN_d^*}{dh}\frac{h}{N_d^*}dh\right) = \frac{-k\xi_d}{(\gamma-1)^2}\left(1+\frac{2\Lambda_{\mathrm{m}}+2\Lambda_{\mathrm{x}}}{1+2\Lambda_{\mathrm{m}}+2\Lambda_{\mathrm{x}}}\right)dh \tag{C.12}$$

Eq.(C.12) indicates a non-monotonic relationship between ξ and γ. For $\gamma < 1$, the stronger the degree of inter-temporal spillover (high γ) implies a larger elasticity of the steady-state number of varieties with respect to a reduction in trade costs, which implies that a greater welfare loss (loss in variety) due to a reduction in trade costs. □

C.3 Proof of Proposition (3)

Proof (1) With exogenous spillover:

$$\frac{dy}{dh}\frac{h}{y}dh = k\xi_d\left[1+\frac{1}{\beta}-\frac{1}{1-\gamma}\right]dh \tag{C.13}$$

One can see that reduction in variable trade cost improves y if and only if $\gamma < \frac{1}{1+\beta}$. Letting $f_m \to \infty$, (C.13) reduces to:

$$\frac{dy}{dh}\frac{h}{y}dh = k\xi_d^T\left[1+\frac{1}{\beta}-\frac{1}{1-\gamma}\right]dh \tag{C.14}$$

We show that $\xi_d < \xi_d^T$, therefore, it is easy to see that in the presence of MP the elasticity of steady-state per-capita output with respect to change in trade cost is lower, which lowers welfare gain from trade as long as $\gamma < \frac{\sigma-1}{k+\sigma-1}$.

(2) In endogenous spillover case i.e. $\Upsilon = \frac{\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}}}{1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}}}$,

$$\frac{dy}{dh}\frac{h}{y}dh = k\xi_d\left\{\underbrace{1+\frac{1}{\beta}}_{\text{Static Effect}} - \underbrace{\frac{1}{1-\gamma}\left[1-\frac{\gamma}{1+2\Lambda_x+2\Lambda_m}\right]}_{\text{Dynamic Effect}}\right\}dh \tag{C.15}$$

The welfare effect of change in variable trade cost can be decomposed into two parts: one representing the static effect and the other representing the dynamic effect. The static effect dominate the dynamic negative effect if $\gamma < \frac{1}{1+\beta\left(1-\frac{1}{1+2\Lambda_x+2\Lambda_m}\right)}$ or $\gamma < \frac{1}{1+\beta\left(1-\frac{1}{\vee}\right)}$.

For example, if $\gamma = 0$:

$$\frac{dy}{dh}\frac{h}{y} = \xi_d(\sigma-1) > 0 \text{ for } \ell > 1 \text{ and } \beta > 1 \tag{C.16}$$

Letting $f_m \to \infty$,

$$\frac{dy}{dh}\frac{h}{y} = k\xi_d^T\left[1+\frac{1}{\beta}-\frac{1}{1-\gamma}\left(1-\frac{\gamma}{1+2\Lambda_x}\right)\right] \tag{C.17}$$

Comparing (C.15) and (C.15) also suggest that the elasticity of welfare with respect to change in trade cost is lower in the presence of MP. □

Figure 5 about here

C.4 Proof of Proposition (4)

Proof (a) In order to establish this proposition, first we obtain [45]

$$\frac{\partial \Lambda_x}{\partial k} = -\frac{\Lambda_x}{(\sigma-1)} \ln(\frac{f_x}{f_d h}) \equiv -\frac{\Lambda_x}{(\sigma-1)} \ln \epsilon^{\sigma-1} < 0 \tag{C.18}$$

$$\frac{\partial \Lambda_m}{\partial k} = -\frac{\Lambda_m}{(\sigma-1)} \ln(\frac{f_m - f_x}{f_d(1-h)}) = -\frac{\Lambda_m}{(\sigma-1)} \ln{(\epsilon\ell)}^{\sigma-1} < 0 \tag{C.19}$$

$$\frac{\partial(\Lambda_x+\Lambda_m)}{\partial k} = -\frac{\Lambda_x}{(\sigma-1)} \ln \epsilon^{\sigma-1} - \frac{\Lambda_m}{(\sigma-1)} \ln{(\epsilon\ell)}^{\sigma-1} \equiv -\frac{\ln \epsilon^{\sigma-1}\Lambda_x}{(\sigma-1)}\left(1+\frac{\Lambda_m}{\Lambda_x}\ln \ell^{\sigma-1}\right) < 0 \tag{C.20}$$

Differentiating ϕ_d^* with respect to k yields:

$$\begin{aligned} \frac{d\ln(\phi_d^*/\underline{\phi})}{dk} &= -k^{-2}\ln\left[\frac{f_d}{f_e(\beta-1)}\left\{1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}}\right\}\right] - \frac{k^{-1}}{(k-(\sigma-1))} + \\ &\quad \frac{k^{-1}}{(1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}})(\sigma-1)}\left\{-\Lambda_{\mathrm{x}}\ln(\frac{f_x}{f_d h}) - \Lambda_{\mathrm{m}}\ln(\frac{f_m-f_x}{f_d(1-h)})\right\} \equiv \xi_k \end{aligned} \tag{C.21}$$

Simplifying Eq.(C.21) using the (C.18)–(C.20) gives:

$$\begin{aligned} \frac{d\ln(\phi_d^*/\underline{\phi})}{dk} &= -k^{-2}\ln\left[\frac{f_d}{f_e(\beta-1)}\left\{1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}}\right\}\right] - \frac{k^{-1}}{(k-(\sigma-1))} \\ &\quad -\frac{k^{-1}}{(1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}})(\sigma-1)}\left\{\ln \epsilon^{\sigma-1}\left(\Lambda_x+\Lambda_{\mathrm{m}}\ln \ell^{\sigma-1}\right)\right\} \equiv \xi_k < 0 \end{aligned} \tag{C.22}$$

after some simplification we get (67) in the main text.

With exogenous spillover, differentiating the steady-state level of y with respect to k gives:

$$\frac{d\ln y}{dk} = \left\{ \underbrace{\frac{1}{k} + (k+\sigma-1)\xi_k}_{\text{Static Effect}} - \underbrace{\frac{1}{1-\gamma}\left(\frac{1}{k}+k\xi_k\right)}_{\text{Dynamic Effect}} \right\}$$

which can be written as:

$$\frac{d\ln y}{dk} = \frac{1}{k}\left(1-\frac{1}{1-\gamma}\right) + k\xi_k\left(1+\frac{\sigma-1}{k}-\frac{1}{1-\gamma}\right) \lesseqgtr 0$$

When technology spillover is exogenous, greater productivity dispersion (lower k) implies larger y only if $k < (\sigma-1)\left(\frac{1}{\gamma}-1\right)$. This implies for a given inter-temporal spillover, the degree of productivity dispersion must be substantially high.

[45] We use $\partial(a^b)/\partial b = a^b \ln(a)$.

(b) Now we move on examining the link between firm heterogeneity and aggregate welfare gain (loss) from trade. Differentiating (C.1) with respect to k gives:

$$\frac{d}{dk}\left(\frac{\partial \phi_d^*}{\partial h}\frac{h}{\phi_d^*}\right)=\frac{1}{(1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}})}\left\{\frac{\partial}{\partial k}\left(\Lambda_{\mathrm{x}}-\frac{h}{1-h}\Lambda_m\right)-\frac{\partial\left(1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}}\right)}{\partial k}\xi_d\right\} \tag{C.23}$$

Substituting (C.18), (C.19) and (C.20) in (C.23) gives:

$$=\frac{\ln\epsilon^{\sigma-1}\Lambda_x}{(1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}})\,(\sigma-1)}\left\{\frac{1}{\sigma-1}\left(\frac{\Lambda_m}{\Lambda_x}\left(\frac{h}{1-h}\right)\ln\ell^{\sigma-1}-1\right)+\left(1+\frac{\Lambda_m}{\Lambda_x}\ln\ell^{\sigma-1}\right)\xi_d\right\}$$

$$=\frac{\ln\epsilon^{\sigma-1}\Lambda_x}{(1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}})\,(\sigma-1)}\left\{\left(\frac{\Lambda_m}{\Lambda_x}\left(\xi_d+\frac{1}{\sigma-1}\left(\frac{h}{1-h}\right)\right)\right)\ln\ell^{\sigma-1}+(\xi_d-\frac{1}{\sigma-1})\right)\right\}$$

$$=\frac{\Lambda_x\ln\epsilon^{\sigma-1}}{(1+\Lambda_{\mathrm{x}}+\Lambda_{\mathrm{m}})\,(\sigma-1)}\left\{\left(\left(\frac{1-h}{h}\right)\xi_d+\frac{1}{\sigma-1}\right)\ell^{\sigma-1-k}\ln\ell^{\sigma-1}+\left(\xi_d-\frac{1}{\sigma-1}\right)\right\} \tag{C.24}$$

$$=\frac{\xi_d}{1-\ell^{\sigma-1-k}}\left\{\left(\left(\frac{1-h}{h}\right)\xi_d+\frac{1}{\sigma-1}\right)\ell^{\sigma-1-k}\ln\ell^{\sigma-1}+\left(\xi_d-\frac{1}{\sigma-1}\right)\right\} \tag{C.25}$$

$$\frac{\xi_d\ln\epsilon^{\sigma-1}(1-\xi_d)}{(1-\ell^{\sigma-1-k})}\left\{\left(\frac{\xi_d}{(1-\xi_d)h}+1\right)\ell^{\sigma-1-k}\ln\ell^{\sigma-1}-1\right\}$$

Elasticity of domestic cutoff with respect to h can be re-written as:

$$\frac{\xi_d}{(1-\xi_d)h}=\frac{1-\ell^{\sigma-1-k}}{\epsilon^{k-(\sigma-1)}+\ell^{\sigma-1-k}}$$

Substituting in the above equation:

$$=\frac{\xi_d(1-\xi_d)\ln\epsilon^{\sigma-1}}{(1-\ell^{\sigma-1-k})}\left(\frac{1+\epsilon^{k-(\sigma-1)}}{1+(\epsilon\ell)^{k-(\sigma-1)}}\right)\left\{\ln\ell^{\sigma-1}-\frac{1+(\epsilon\ell)^{k-(\sigma-1)}}{1+\epsilon^{k-(\sigma-1)}}\right\}$$

This implies that an increase in firm heterogeneity (lower k) lead to a larger elasticity of the domestic productivity cutoff with respect to reductions in variable trade costs if the term in the curly-bracket is less than zero, i.e. $\ln\ell^{\sigma-1}-\frac{1+(\epsilon\ell)^{k-(\sigma-1)}}{1+\epsilon^{k-(\sigma-1)}}<0$.

Differentiating (C.2) with respect to k or letting $f_m\to\infty$ in (C.24), gives the effects of dispersion on domestic productivity cutoff in the absence of MP:

$$\frac{d}{dk}\left(\frac{\partial\phi_T^*}{\partial h}\frac{h}{\phi_T^*}\right)=\frac{\Lambda_x\left(\xi_d^T-1\right)\ln\epsilon^{\sigma-1}}{(1+\Lambda_{\mathrm{x}})\,(\sigma-1)}<0$$

Therefore, in the absence of MP, greater heterogeneity of firm productivity (smaller k) implies a larger elasticity of the domestic productivity cutoff with respect to reductions in variable trade cost.[46]

□

C.5 Proof of Proposition (5)

Proof

[46] This is identical to the result obtained by Melitz and Redding (2013).

With exogenous spillover: Differentiating (55) after substituting (57) for F :

$$\frac{\partial g}{\partial h}\frac{h}{g}dh = -k\xi_d dh \tag{C.26}$$

With endogenous spillover: Differentiating (55) after substituting (57) and (64a) for F and Υ:

$$\frac{\partial g}{\partial h}\frac{h}{g}dh = -k\xi_d\left(1 - \frac{1}{1+2\Lambda_x+2\Lambda_m}\right)dh \tag{C.27}$$

Comparing this with Proposition C.2 suggest that the level and growth effects of change in trade cost is qualitatively the same.

In the absence of MP, once can find the growth are as

$$\frac{\partial g}{\partial h}\frac{h}{g}dh = -k\xi_d^T\left(1 - \frac{1}{1+2\Lambda_x}\right)dh \tag{C.28}$$

□

D Derivation of Social Optimum

To obtain the socially optimal path, we first solve a static problem of allocating resources to producing various existing goods. Then we solve the the dynamic problem of determining the growth of N_d over time.

Given the total investment, I, the static problem for the central planner is:

$$\max\ \text{K}{=}\left[\int_\Omega q(\phi)^\alpha d\phi\right]^{\frac{1}{\alpha}} \text{ subject to } \int \tfrac{q(\phi)}{\phi}N_d\mu(\phi)d\phi - I \tag{D.1}$$

The Lagrangian of the above optimization problem is:

$$\mathcal{L} = \left[\int_\Omega q(\phi)^\alpha d\phi\right]^{\frac{1}{\alpha}} + \lambda\left[I - \int \frac{q(\phi)}{\phi}N_d\mu(\phi)d\phi\right] \tag{D.2}$$

From the first order condition, it is straightforward to show that:

$$\frac{q(\phi)}{q(\phi')} = \left(\frac{\phi}{\phi'}\right)^\sigma \tag{D.3}$$

we know that $I = \alpha(\alpha Y_t) = \alpha N_d \frac{q(\tilde{\phi})}{\tilde{\phi}}$

$$K = N_d^{\frac{1}{\alpha}} q(\tilde{\phi}) = N_d^{(1-\alpha)/\alpha}\tilde{\phi}I_t \tag{D.4}$$

Therefore, Y , can be written as:

$$Y = AL_Y^{1-\alpha}K^\alpha = AL_Y^{1-\alpha}N_d^{1-\alpha}(\tilde{\phi}I)^\alpha \tag{D.5}$$

From the above equation, Y , is expressed as a function of L_Y, N_d, $\tilde{\phi}$ and I .

In the dynamic problem, the central planner maximizes:

$$\max U = \int_0^\infty \left(\frac{C_t^{1-\theta}-1}{1-\theta}\right)e^{-\rho t}dt \tag{D.6}$$

The social planner maximizes (D.6) subject to (D.7) and (D.8)

$$\dot{N}_d = \frac{(L - L_Y)^\lambda}{L^\upsilon} \frac{{N_d}^\gamma}{\mathrm{F}} \tag{D.7}$$

$$0 = Y - C - I \tag{D.8}$$

$$L_I + L_Y = L \tag{D.9}$$

$$\frac{\dot{L}}{L} = n \tag{D.10}$$

The corresponding present-value Hamiltonian is given by:

$$\mathcal{H} = \frac{\left(C^{1-\theta} - 1\right) e^{-\rho t}}{1-\theta} + \mu_K \left(A L_Y^{1-\alpha} N_d^{1-\alpha} (\tilde{\phi} I)^\alpha - C - I\right) + \mu_N \left(\frac{(L - L_Y)^\lambda}{L^\upsilon} \frac{{N_d}^\gamma}{\mathrm{F}}\right) \tag{D.11}$$

where C , L_Y and I are control variables and N_d is a state variable.
Necessary optimal conditions are $\partial\mathcal{H}/\partial C = \partial\mathcal{H}/\partial L_Y = \partial\mathcal{H}/\partial I = 0$,

$$\frac{\partial \mathcal{H}}{\partial C_t} = C^{-\theta} e^{-t\rho} - \mu_K = 0$$

$$\frac{\dot{\mu}_K}{\mu_K} = -\theta \frac{\dot{C}}{C} - \rho \tag{D.12}$$

$$\frac{\partial \mathcal{H}}{\partial I_t} = 0 \implies I = A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \left(\frac{k - (\sigma - 1)}{k}\right)^{\frac{\alpha}{(-1+\alpha)(-1+\sigma)}} (\phi^*)^{\frac{\alpha}{1-\alpha}} L_Y N_d$$

$$\frac{\dot{I}}{I} = \frac{\dot{L_Y}}{L_Y} + \frac{\dot{N}_d}{N_d} \tag{D.13}$$

From (D.5) that (D.8) one can easily see that Y and C are also growing at the same rate in steady-state.

$$\frac{\dot{C}}{C} = \frac{\dot{L_Y}}{L_Y} + \frac{\dot{N}_d}{N_d} \tag{D.14}$$

$$\frac{\partial \mathcal{H}}{\partial L_Y} = \frac{A^{\frac{1}{-1+\alpha}} L^{-\upsilon} (L - L_Y)^{\lambda - 1} {N_d}^{\gamma - 1} \alpha^{\frac{\alpha}{-1+\alpha}} \lambda \mu_N \left(\frac{k}{1+k-\sigma}\right)^{\frac{\alpha}{(-1+\alpha)(-1+\sigma)}} (\phi^*)^{\frac{\alpha}{-1+\alpha}}}{\mathrm{F}(1-\alpha)} - \mu_K = 0 \tag{D.15}$$

$$\frac{\mu_K}{\mu_N} = \frac{A^{\frac{1}{-1+\alpha}} L^{-\upsilon} (L - L_Y)^{\lambda - 1} {N_d}^{\gamma - 1} \alpha^{\frac{\alpha}{-1+\alpha}} \lambda \left(\frac{k}{1+k-\sigma}\right)^{\frac{\alpha}{(-1+\alpha)(-1+\sigma)}} (\phi^*)^{\frac{\alpha}{-1+\alpha}}}{\mathrm{F}(1-\alpha)} \tag{D.16}$$

Assuming F and ϕ^* constant in steady state and L_Y and L grow at an exogenous rate n.

$$\frac{\dot{\mu}_K}{\mu_K} = (\lambda - \upsilon - 1) n + \frac{\dot{\mu}_N}{\mu_N} + (\gamma - 1) \frac{\dot{N}_d}{N_d} \tag{D.17}$$

Now to the dynamic constraint:

$$\frac{\partial \mathcal{H}}{\partial N_d} = \dot{\mu_N} = -\frac{L^{-\upsilon} L_I^{\lambda} N_d^{\gamma-1} \gamma \mu_N}{\mathrm{F}} - A I^{\alpha} L_Y^{1-\alpha} N_d^{-\alpha} (1-\alpha) \mu_K \left(\left(\frac{k}{1+k-\sigma} \right)^{\frac{1}{\sigma-1}} \phi^* \right)^{\alpha} \tag{D.18}$$

$$\frac{\dot{\mu}_N}{\mu_N} = -\frac{L^{-\upsilon} L_I^{\lambda} N_d^{\gamma-1} \gamma}{\mathrm{F}} - A I^{\alpha} L_Y^{1-\alpha} N_d^{-\alpha} (1-\alpha) \frac{\mu_K}{\mu_N} \left(\left(\frac{k}{1+k-\sigma} \right)^{\frac{1}{\sigma-1}} \phi^* \right)^{\alpha} \tag{D.19}$$

Substituting (D.16) into (D.19) yields:

$$\frac{\dot{\mu}_N}{\mu_N} = -\frac{L^{-\upsilon} L_I{}^{\lambda-1} N^{\gamma-1} (L_I \gamma + L_Y \lambda)}{\mathrm{F}} = -\frac{\dot{N}_d}{N_d} \left(\gamma + \left(\frac{1-S_I}{S_I} \right) \lambda \right) \tag{D.20}$$

Solving (D.17) and (D.12) together with (D.14) yields:

$$\frac{\dot{N}_d}{N_d} = -\frac{\frac{\dot{\mu}_N}{\mu_N} + \rho + n(\lambda - \upsilon - 1)}{-1 + \gamma + \theta} \tag{D.21}$$

Substituting (D.20) into (D.21) gives:

$$\frac{\dot{N}_d}{N_d} = \frac{1}{\gamma + \theta - 1} \left\{ \frac{\dot{N}_d}{N_d} \left(\gamma + \left(\frac{1-S_I}{S_I} \right) \lambda \right) - (\rho + n(\lambda - \upsilon - 1)) \right\} \tag{D.22}$$

Now we can find the steady-state growth rate for social planner by distinguishing the two classes of models that we discussed earlier.

When $\gamma < 1$: The steady-state growth rate when $\gamma < 1$ can be obtained from (D.22):

$$g_S = \frac{(\lambda - \upsilon) n}{1 - \gamma} \tag{D.23}$$

By plugging this in (D.21) yields:

$$\frac{\dot{\mu}_N}{\mu_N} = -\theta g_S - (\rho - n) \tag{D.24}$$

from (D.20)

$$\frac{\dot{\mu}_N}{\mu_N} = -g_S(\gamma + \frac{1 - s_I}{s_I} \lambda) \tag{D.25}$$

Solving (D.24) and (D.25) simultaneously gives the socially optimal share of labor:

$$S_I = \frac{1}{1 + \frac{1}{\lambda} \left[\frac{\rho}{g_S} + \theta - \gamma \right]} \tag{D.26}$$

When $\gamma = 1$: Now we turn to our special case in which $\gamma = 1$. Implementing the assumptions γ in (D.20) yields: $\frac{\dot{\mu}_N}{\mu_N} = -\frac{L^{1-\upsilon}}{F}$. Plugging this value in (D.21) gives the growth rate under social planner.

$$\frac{\dot{N}_d}{N_d} = \frac{1}{\theta} \left[\frac{L^{1-\upsilon}}{F} - (\rho - n) \right] \tag{D.27}$$

Implementing the assumptions $\gamma = \lambda = 1$ and $\upsilon = 0$ in (D.20) yields:

$$\frac{\dot{N}_d}{N_d} = \frac{1}{\theta}\left[\frac{L}{F} - (\rho - n)\right]$$

which exhibits strong scale-effect. Setting $\upsilon = 1$ provides scale-free endogenous growth rate:

$$\frac{\dot{N}_d}{N_d} = \frac{1}{\theta}\left[\frac{1}{\mathrm{F}} - (\rho - n))\right] \tag{D.28}$$

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E Decentralized Equilibrium

E.1 Deriving the dynamic optimization 3

Household asset accumulates according to the following function:

$$\dot{A}_t = r_t A_t + w_t L_t - C_t \tag{E.1}$$

where A_t is household asset at time t , r_t denotes the interest rate, w_t is real wage rate, and C_t is consumption of final good.

Dividing (E.1) by L_t and denoting per capita asset by a_t gives:

$$\frac{\dot{A}_t}{L_t} = r_t a_t + w_t - c_t \tag{E.2}$$

c_t is the amount spend per household member.

Differentiating per capita asset $a_t = \frac{A_t}{L_t}$ w.r.t. time gives:

$$\dot{a}_t = \frac{\dot{A}_t L_t - A_t \dot{L}}{L^2} = \frac{\dot{A}_t}{L_t} - n a_t$$

The per-capita budget constraint can be obtained by plugging the above equation in (E.2):

$$\dot{a}_t = (r_t - n)a_t + w_t - c_t$$

The household optimization problem is given as:

$$\max U = \int_0^\infty \left(\frac{c_t^{1-\theta} - 1}{1-\theta} \right) e^{-(\rho-n)t} dt$$

$$\text{subject to: } \dot{a}_t = (r_t - n)a_t + w_t - c_t$$

According to (E.1) and (E.2), the present value Hamiltonian is given by:

$$\mathcal{H} = e^{-(\rho-n)t} \left(\frac{c^{1-\theta} - 1}{1-\theta} \right) + \mu_t[(r_t - n)a_t + w_t - c_t] \tag{E.3}$$

where μ is a co-state-variable associated with constraint (E.2). Here $c_t$is the control variable and a_t is the state variable. The necessary optimal conditions are:

$$\frac{\partial \mathcal{H}}{\partial c_t} = e^{-(\rho-n)t} c_t^{-\theta} - \mu_t = 0 \tag{E.4}$$

$$\frac{\partial \mathcal{H}}{\partial a_t} = \mu_t(r_t - n) = -\dot{\mu}_t \tag{E.5}$$

Together with the transversality condition:

$$\lim_{t-\infty} e^{-(\rho-n)t} \mu_t a_t = 0$$

Log differentiating (E.4) w.r.t. time yields:

$$\frac{\dot{\mu}_t}{\mu} = n - \rho - \theta \frac{\dot{c}_t}{c_t}$$

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From (E.5), we get $\frac{\dot{\mu}_t}{\mu} = n - r_t$.

Equating (E.5) and (**??**) give the differential equation in (3) in the main text.

E.2 Derivation of demand function 10

A firm solves the following maximization problem:

$$\max_{q^c} K = \left[\sum_{c=H,F} \int_{\Omega^c} q^c(\phi)^\alpha dG^c(\phi) \right]^{\frac{1}{\alpha}} \text{ subject to } \textstyle\sum_{c=H,F} \int_{\Omega^c} p^c(\phi) q^c(\phi) dG^c(\phi) = \alpha Y \tag{E.6}$$

Setting up the Lagrangian:

$$\mathcal{L} = \left[\sum_{c=H,F} \int_{\Omega^c} q^c(\phi)^\alpha dG^c(\phi) \right]^{\frac{1}{\alpha}} + \lambda [\alpha Y - \textstyle\sum_{c=H,F} \int_{\Omega^c} p^c(\phi) q^c(\phi) dG^c(\phi)]$$

First order condition with respect to any $q(\phi)$ is:

$$\frac{\partial \mathcal{L}}{\partial q^c(\phi)} = K^{1-\alpha} q^c(\phi)^{\alpha-1} - \lambda p^c(\phi) = 0 \tag{E.7}$$

By simplifying and rearranging this equation gives:

$p^c(\phi) = (\lambda p^c(\phi))^{\frac{1}{\alpha-1}} K$

Take first order condition with respect to $q(\phi)$ and with respect to $q(\phi')$ and divide one by the other. This yields:

$$q^c(\phi) = \left(\frac{p^c(\phi')}{p^c(\phi)} \right)^{\frac{1}{1-\alpha}} q^c(\phi') \tag{E.8}$$

Plugging this into budget constraint:

$$\sum_{c=H,F} \int_{\Omega^c} p^c(\phi) \left(\frac{p^c(\phi')}{p^c(\phi)} \right)^{\frac{1}{1-\alpha}} q^c(\phi') dG^c(\phi) = \alpha Y_t$$

$$q^c(\phi') = \frac{\alpha Y_t}{p^c(\phi')^{\frac{1}{1-\alpha}} \sum_{c=H,F} \int_{\Omega^c} p^c(\phi)^{\frac{-\alpha}{1-\alpha}} dG^c(\phi)} \tag{E.9}$$

By plugging (E.9) in (E.8) yields:

$$q^c(\phi) = \frac{\alpha Y_t}{p(\phi)^{\frac{1}{1-\alpha}} \sum_{c=H,F} \int p(\phi)^{\frac{-\alpha}{1-\alpha}} dG^c(\phi)} \equiv \frac{\alpha Y_{it} p_i^c(\phi)^\sigma}{\sum_{c=H,F} \int_{\Omega_i^c} p_i^c(\phi)^{1-\sigma} dG^c(\phi)}$$

using the definition of P_t from (7) gives the demand function (10) in the main text.

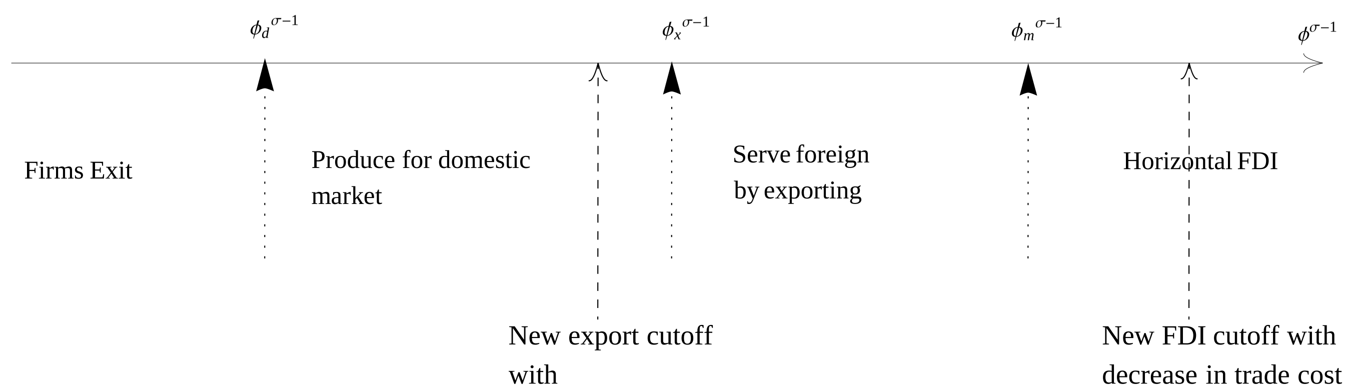

Figure 1: Effects of decrease in trade costs on productivity cutoff

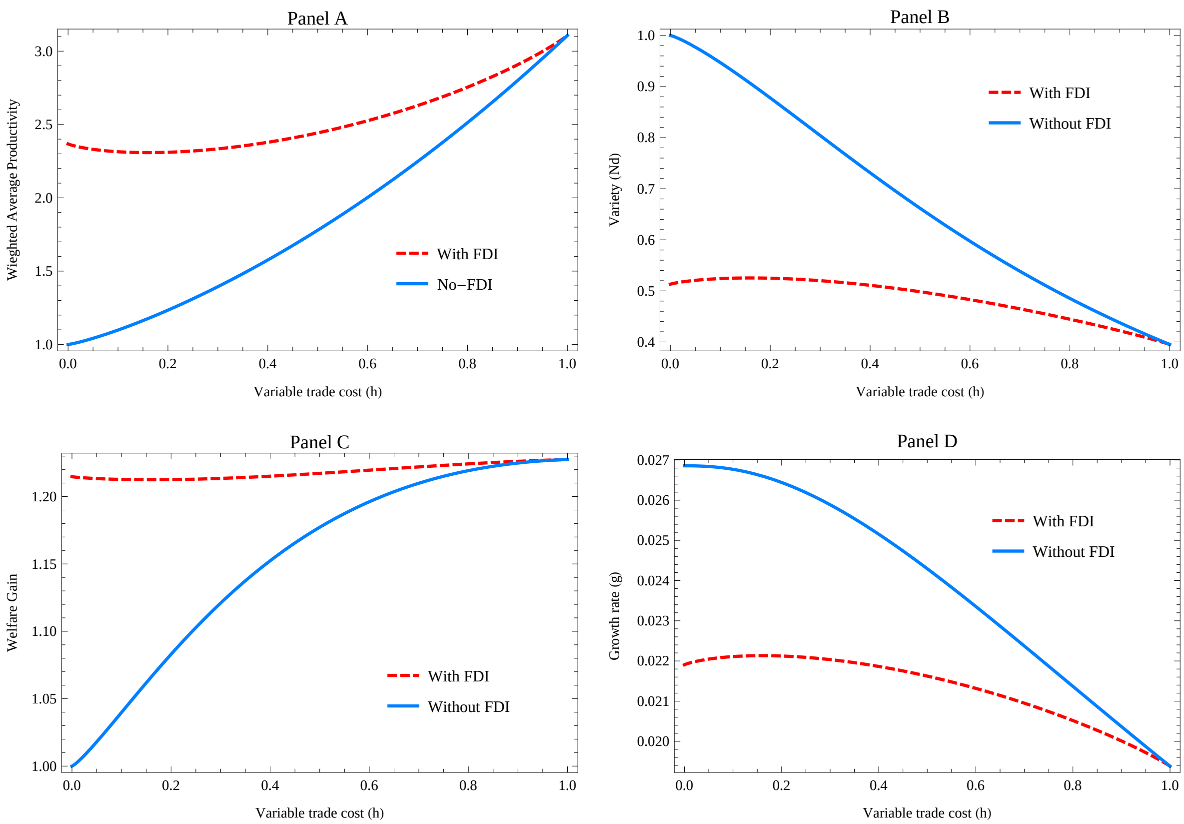

Figure 2: Decrease in variable trade cost

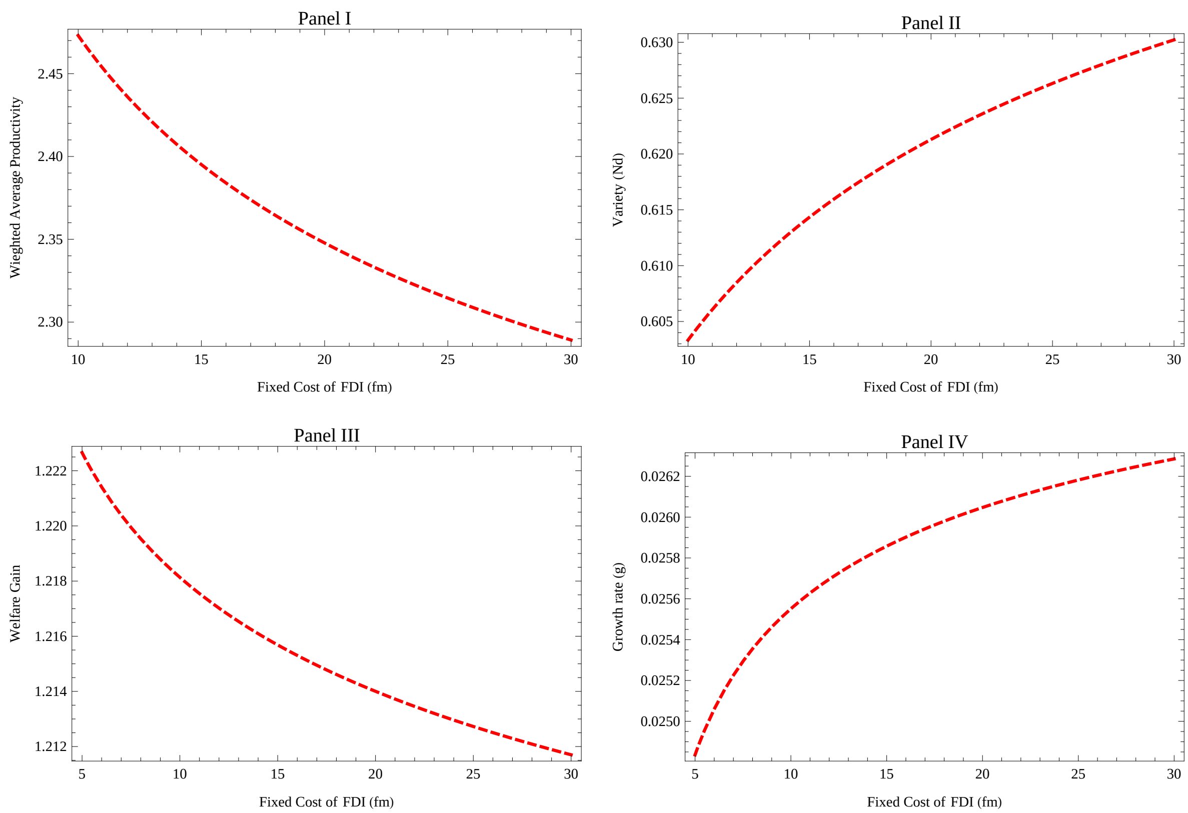

Figure 3: Reduction of FDI fixed costs

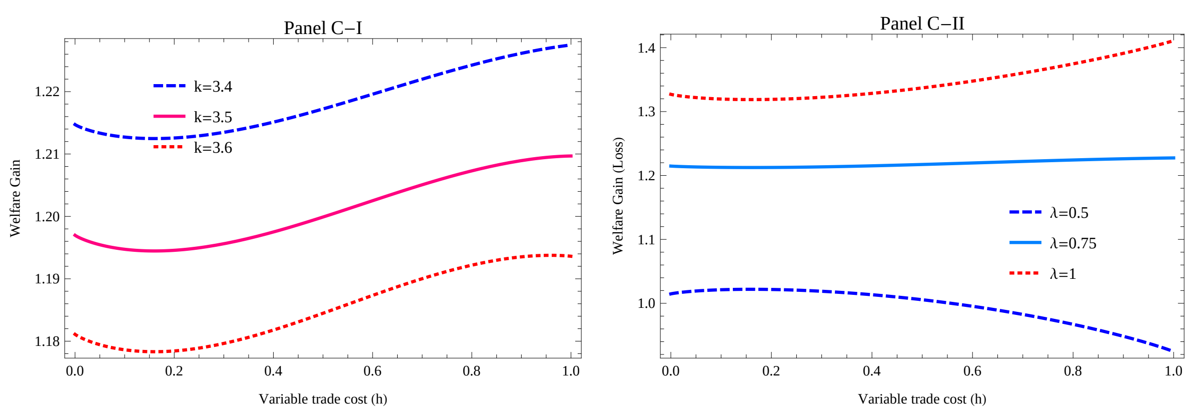

Figure 4: Heterogeneity, R&D Duplication and Welfare

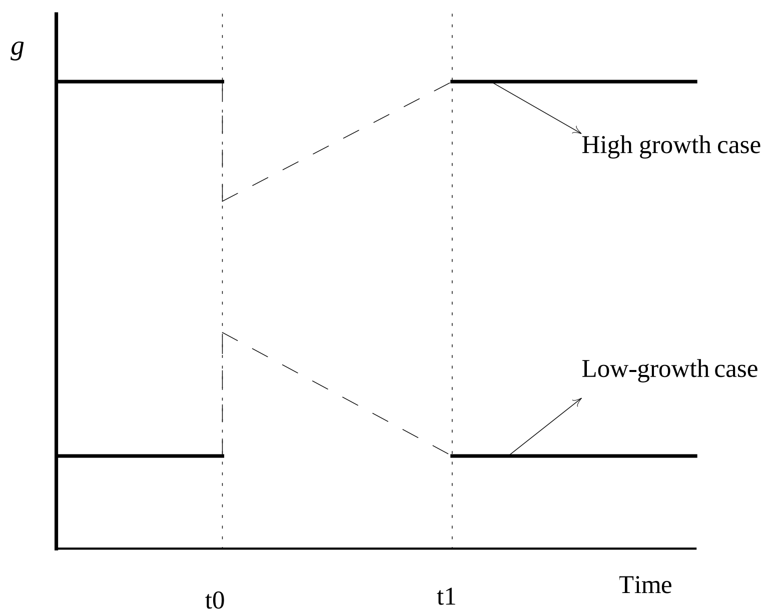

Figure 5: Growth rate along the transition path in response to a reduction in trade cost for high-growth i.e. $\gamma > \frac{1}{1+\beta}$ or $\gamma > \frac{1}{1+\beta\left(1-\frac{1}{\triangledown}\right)}$ and low-growth i.e. $\gamma < \frac{1}{1+\beta}$ or $\gamma < \frac{1}{1+\beta\left(1-\frac{1}{\triangledown}\right)}$ case for $\ell > 1$

Table 3: Summary of the results

Static-Component $(\frac{\partial\Phi}{\partial h}\frac{h}{\Phi})$	Dynamic Component $(\frac{dN_d^*}{dh}\frac{h}{N_d^*})$		Aggregate Welfare $(\frac{dy}{dh}\frac{h}{y})$	
	Exogenous	Endogenous	Exogenous	Endogenous
Trade and FDI Models				
$=(k+\sigma-1)\xi_d$	$=-\frac{k}{1-\gamma}\xi_d<0$	$=-\frac{k}{1-\gamma}\xi_d\left[1-\frac{\gamma}{1+2\Lambda_m+2\Lambda_x}\right]$	$=k\xi_d\left[1+\frac{1}{\beta}-\frac{1}{1-\gamma}\right]$	$=k\xi_d\left\{\underbrace{1+\frac{1}{\beta}}_{\text{Static Effect}}\underbrace{-\frac{1}{1-\gamma}\left[1-\frac{\gamma}{1+2\Lambda_x+2\Lambda_m}\right]}_{\text{Dynamic Effect}}\right\}$
where $\frac{\partial\phi_d^*}{\partial h}\frac{h}{\phi_d^*}=\frac{\Lambda_x(1-\ell^{\sigma-1-k})}{(\sigma-1)(1+\Lambda_x+\Lambda_m)}\equiv\xi_d>0$ for $\ell>1$				
Trade Only Models				
$=(k+\sigma-1)\xi_d^T$	$=-\frac{k}{1-\gamma}\xi_d^T<0$	$=-\frac{k}{1-\gamma}\xi_d^T\left[1-\frac{\gamma}{1+2\Lambda_x}\right]$	$=k\xi_d^T\left[1+\frac{1}{\beta}-\frac{1}{1-\gamma}\right]$	$=k\xi_d^T\left\{\underbrace{1+\frac{1}{\beta}}_{\text{Static Effect}}\underbrace{-\frac{1}{1-\gamma}\left[1-\frac{\gamma}{1+2\Lambda_x}\right]}_{\text{Dynamic Effect}}\right\}$
where $\frac{\partial\phi_T^*}{\partial h}\frac{h}{\phi_T^*}=\frac{\Lambda_x}{(\sigma-1)(1+\Lambda_x)}>0\equiv\xi_d^T>0$				

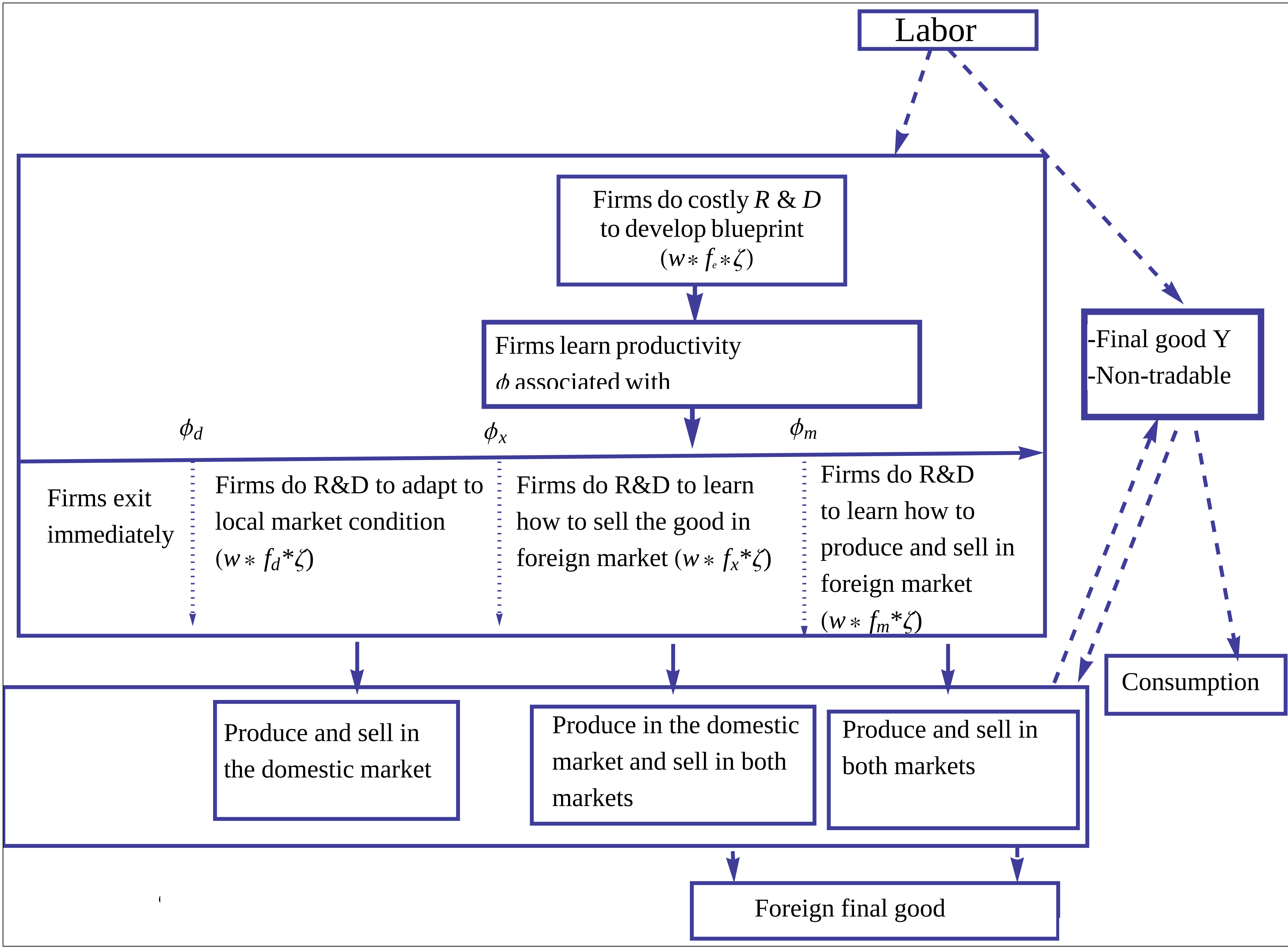

Figure 6: Structure of the model

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